$\qquad$ Student Number: $\qquad$

This exam is closed book/notes/neighbors/etc. Answer all questions on these exam pages. No code or pseudo-code is necessary - just a precise and concise explanation and justification. Unsupported work will receive no credit.

Q1 of 5 (5 pts) Fill in the following table using $\Theta$ notation. While no explanation is required, if you feel it necessary to include an explanation or comment, please provide such information in the space provided below the table.

| Model | Communication <br> Diameter | Bisection Width | Degree of <br> Network |
| :--- | :--- | :--- | :--- |
| Mesh of size $n$ |  |  |  |
| Hypercube of size $n$ |  |  |  |
| $\mathrm{~K}_{n}$ |  |  |  |
| Star of size $n$ |  |  |  |

$\mathrm{K}_{n}$ is to defined to be a set of $n$ processors where every processor $\mathrm{P}_{i}$ is directly connected to every processor $\mathrm{P}_{j}$ for all $1 \leq i \leq n, 1 \leq j \leq n$, and $i \neq j$.

For a definition of a star of size n, please refer to Q2.

NAME: $\qquad$
Q2 of 5 (9 pts) Define a star of size $n$ to be a set of $n$ processors, labeled $P_{0}, P_{1}, \ldots, P_{n-1}$, where processor $P_{0}$ is connected to every processor $\mathrm{P}_{i}, 1 \leq i \leq n-1$. That is, there are only $n$ - 1 communication links in a star of size $n$.

Justify your answers to each of the following.
A. Give an asymptotically optimal algorithm to find the minimum of a set of $n$ pieces of data, initially distributed 1 per processor in an arbitrary fashion.
B. Give an asymptotically optimal algorithm to compute the parallel prefix of a set of $n$ pieces of data, initially distributed 1 per processor in an arbitrary fashion.
C. Give an asymptotically optimal algorithm to sort of a set of $n$ pieces of data, initially distributed 1 per processor in an arbitrary fashion.

NAME:

Q3 of 5 ( 10 pts ) Given $n$ integers arbitrarily and evenly distributed amongst the processors of the following architectures, determine the sum of these $n$ values. Define your architecture such that your solution is asymptotically-optimal and cost-optimal.
i) PRAM
ii) Linear Array
iii) Hypercube
iv) Star

NAME:

Q4 of 5 ( 6 pts ) Given a pyramid of size $n$, with a 0 or 1 stored in every base processor, give an algorithm that will determine for every processor, the number of base processors in its subtree that have a 1 in it. Note that this is equivalent to each processor knowing the sum of the 1 's in the base of its subtree. Discuss the optimality of the running time of your algorithm.

NAME:

Extra Credit (1 pt) From what institution did Prof. Miller receive his Ph.D.?

Extra Credit (1 pt) How many books has Prof. Miller co-authored?

