$\qquad$ Student Number: $\qquad$

This exam consists of problems on 2 pages. Make sure you answer the questions on both pages in a blue book. Return this piece of paper with your blue book(s).

Plagiarism will earn you an $F$ in the course and a recommendation of expulsion from the university. You may not refer to any material outside of this exam. That is, you may not refer to notes, books, papers, calculators, phones, classmates, classmates' exams, and so forth. You may use a writing utensil and this exam. Nothing more.

Answer all questions in the blue book(s). No code or pseudo-code is necessary - just a precise and concise explanation and justification. Unsupported work will receive no credit.

Q1 (5 pts) Prove that $\sum_{k=1}^{n} k^{1 / 4}=\Theta\left(n^{5 / 4}\right)$ by the method of approximating an integral by summations of rectangles.

Q2 (5 pts) Given $n$ values stored in the global memory of a CREW PRAM, give a costoptimal algorithm to determine the parallel prefix-sum of the values, which should be stored in the global memory. Running time counts. Justify your result.

Q3 (5 pts) Given $n$ values, evenly distributed amongst the processors of a mesh, give a cost-optimal algorithm to determine the parallel prefix-sum of the values. Running time counts. Justify your result.

Q4 (5 pts) Hypercube question.
a. Draw and label a Hypercube of size 16 .
b. What is the communication diameter of a hypercube of size $n$ ? Justify your answer.
c. Given $n$ values, distributed one per processor on a hypercube of size $n$, give an optimal algorithm to determine the parallel prefix of these values. Justify your result.

Q5 (5 pts) Given one value per leaf processor on a tree with $n$ leaf processors, give an optimal algorithm to determine the parallel prefix-sum of these values. Running time counts. Justify your result.

Q6 (5 pts) Given one integer stored per base processor on a mesh-of-trees of base size $n$, give an efficient algorithm for each of the following.
a. Replace the value in every base processor $P_{i, j}$ with the median of its value and the values in its four nearest mesh neighbors.
b. Replace the value in every base processor $P_{i, j}$ with the sum of the values in row $i$ and the values in column $j$.
$\qquad$ Student Number:

Bonus Question 1 (1 pt). Does Dr. Miller play in a band? (Choose one.)
a) Yes, the CSE House Band.
b) Yes, Escher's Enigma.
c) Yes, Maximal Efficiency.
d) No, he doesn't play music.

Bonus Question 2 (1 pt). Where did Dr. Miller grow up? (Choose one.)
a) Buffalo
b) Binghamton
c) Long Island
d) He hasn't grown up

Bonus Question 3 (1 pt). Dr. Miller's research is in what area? (Choose one.)
a) Machine Learning.
b) Video gaming.
c) Operating systems.
d) Parallel algorithms.

