PARALLEL IMPLEMENTATION OF FLOYD-WARSHALL ALGORITHM

Guided by: Dr. Russ Miller (UB Distinguished Professor)
CSE 633: Parallel Algorithms
Presented By: Asmita Gautam

University at Buffalo
Department of Computer Science and Engineering
School of Engineering and Applied Sciences

OVERVIEW:

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2. Serial Execution
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4. Implementation and Parallel Algorithm
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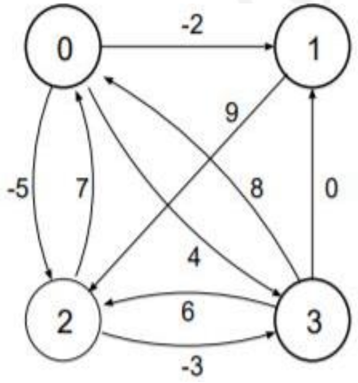
Problem Statement

Perform the parallel implementation of the Floyd-Warshall algorithm.

Floyd-Warshall Algorithm:

- It is an all pair shortest path algorithm for a directed and weighted graph.
- It basically tries to find the minimum distance between any pair of vertices in the graph.
- In this we consider every vertex as an intermediate vertex ‘k’ and find if the distance between i,j through k is smaller than the existing distance.

i.e. dist(i,j) = min(dist(i,j) , dist(i,k) + dist(k,j))



	0	1	2	3
0	0	−2	−5	4
1	∞	0	9	∞
2	7	∞	0	−3
3	8	0	6	0

Adjacency Matrix

Serial Execution

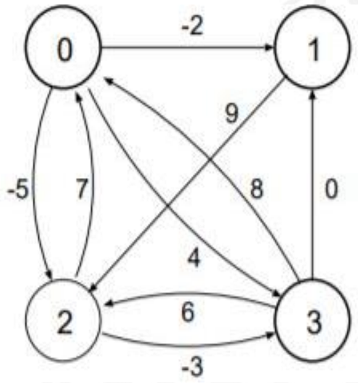
- Since the distance from a vertex to itself is going to be 0 , hence all the diagonals are set to 0 in the matrix.

- **Sequential Algorithm:**

Input: n — number of vertices
a — adjacency matrix
Output: Transformed a that contains the shortest path lengths

for $k \leftarrow$ 0 to $n-1$
  for $i \leftarrow$ 0 to $n-1$
    for $j \leftarrow$ 0 to $n-1$
      $a[i,j] \leftarrow \min(a[i,j],\ a[i,k] + a[k,j])$
    endfor
  endfor
endfor

Easy to see that the algorithm is $\Theta(n^3)$



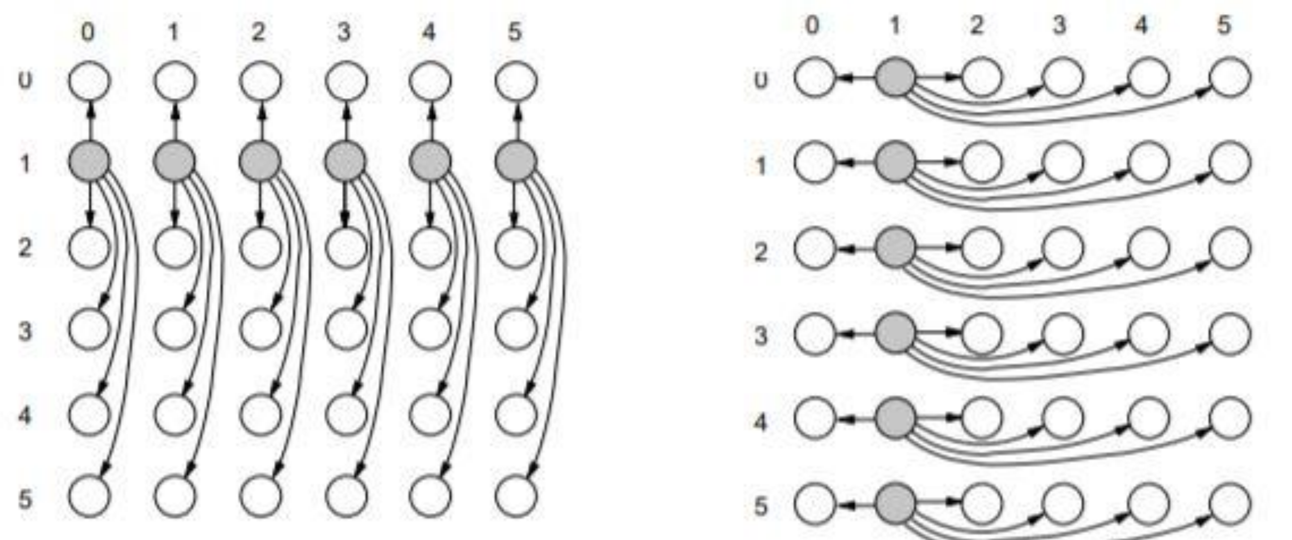
	0	1	2	3
0	0	−2	−5	4
1	∞	0	9	∞
2	7	∞	0	−3
3	8	0	6	0

Adjacency Matrix

Parallelism.. But How?

- We see that the task responsible for updating A[i, j] needs the values of A[i, k] and A[k, j]
- **For k=1**
- the task responsible for A[0, 2] needs access to A[0, 1] and A[1, 2]
- the task responsible for A[1, 2] needs access to A[1, 1] and A[1, 2]
- the task responsible for A[2, 2] needs access to A[2, 1] and A[1, 2]
- the task responsible for A[3, 2] needs access to A[3, 1] and A[1, 2]
- That means for a particular k, j, A[k][j] is needed by all the column elements of j.
- Similarly,
- the task responsible for A[0, 0] needs access to A[0, 1] and A[1, 0]
- the task responsible for A[0, 1] needs access to A[0, 1] and A[1, 1]
- the task responsible for A[0, 2] needs access to A[0, 1] and A[1, 2]
- the task responsible for A[0, 3] needs access to A[0, 1] and A[1, 3];

Communication:



(a) Each task in row 1 broadcasts to tasks in same column.

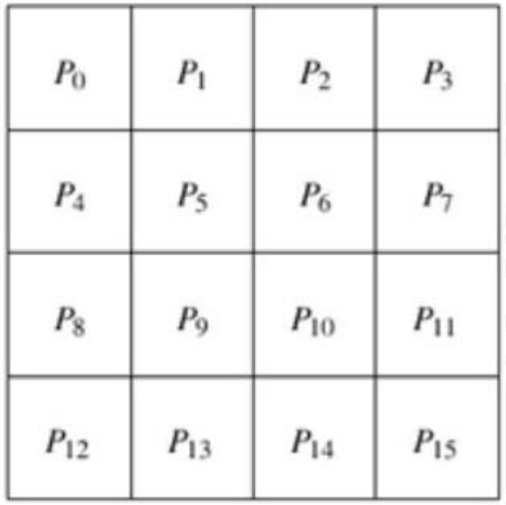
(b) Each task in column 1 broadcasts to tasks in same row.

Here for K = 1
1st column elements would do a respective row broadcast
And 1st row elements would do a respective column broadcast.

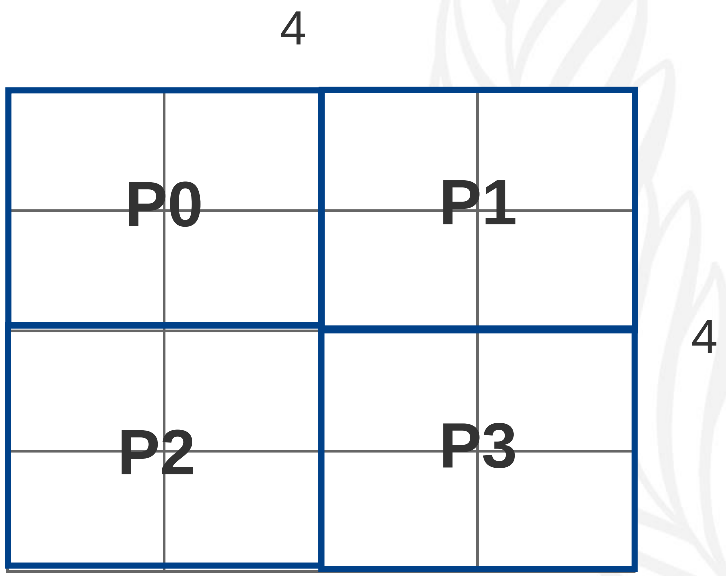
- During iteration k of the outer loop, each element of row k of A must be broadcast to every task in the same column as that element.
- Every element of column k of A must be broadcast to every task in the same row as that element.
- A broadcast is a global communication operation in which a single task sends a message to all processes in its communication group.

Implementation:

- Partitioned the matrix data using 2-D block mapping.
- The entire n x n matrix data is divided into squares of the same size and each square is assigned to a processor.
- For n x n matrix and p processors each process calculates a $n/\sqrt{p} \times n/\sqrt{p}$ part of the distance matrix.



n = 4
n x n = 16 data elements
No of processors = 4
n^2 elements are distributed amongst p processors = $n^2/p = n/\sqrt{p} \times n/\sqrt{p}$

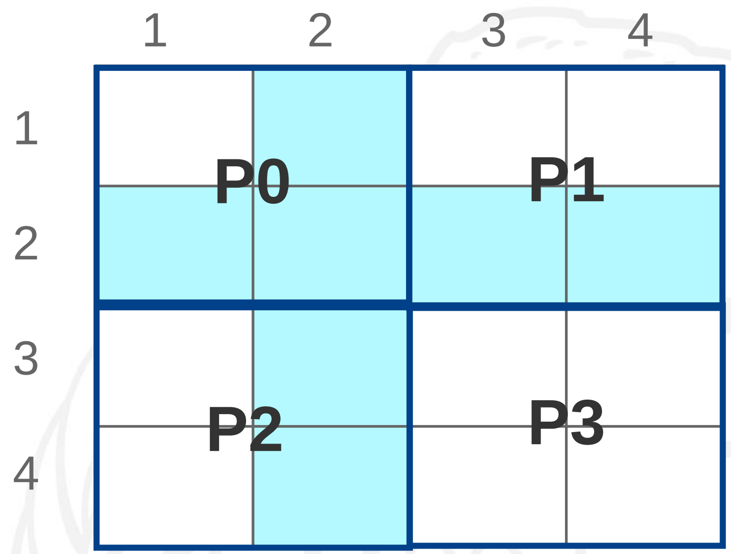


Critical condition for equal distribution of data:

$n \% \sqrt{p} = 0$

Parallel Pseudocode:

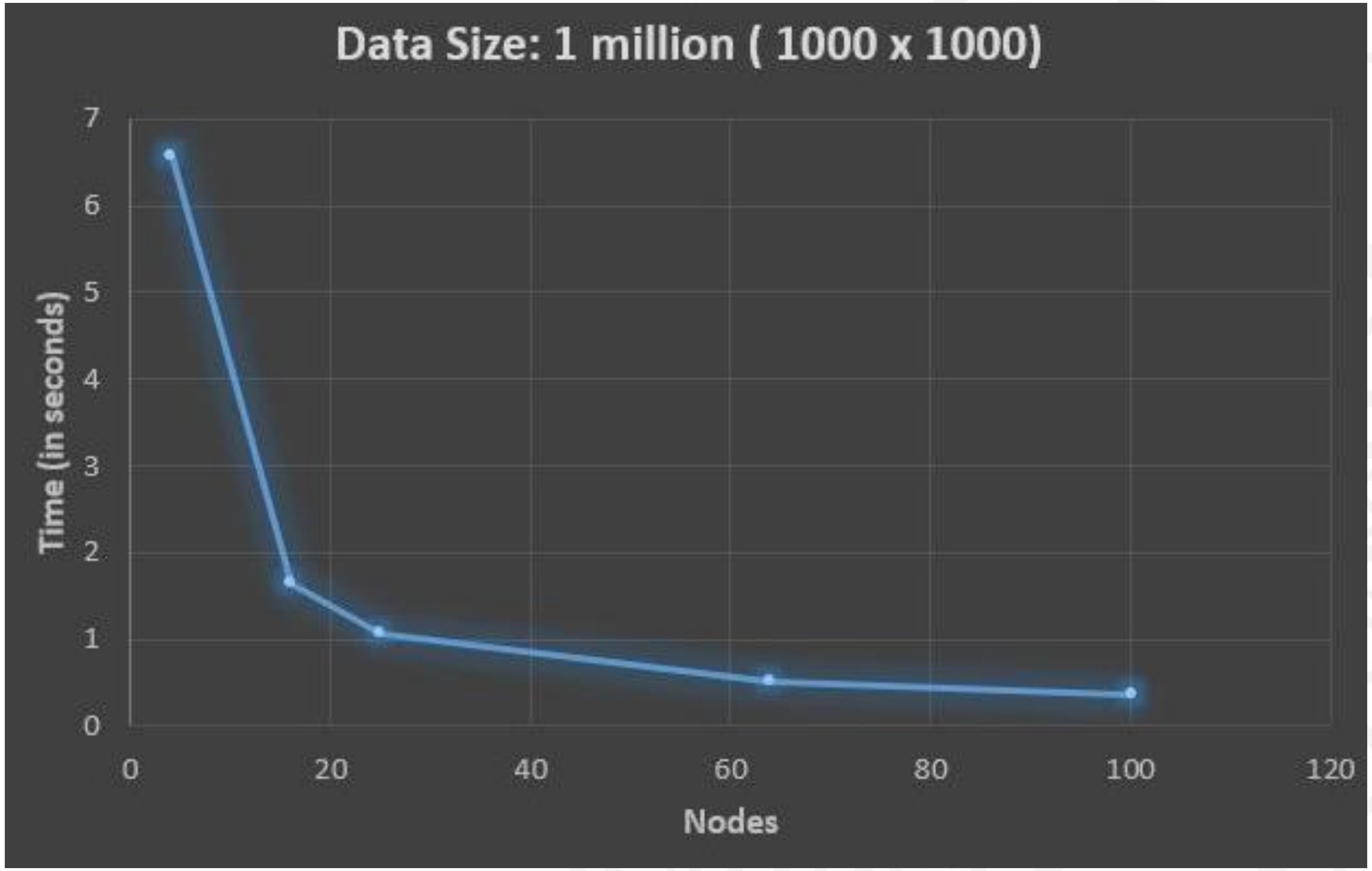
func Floyd_All_Pairs_Parallel($D^{(0)}$) {
  for k := 1 **to** n **do**{
    Each process $p_{i,j}$ that has a segment of the k-th row of $D^{(k-1)}$,
    broadcasts it to the $p_{*,j}$ processes;
    Each process $p_{i,j}$ that has a segment of the k-th column of $D^{(k-1)}$,
    broadcasts it to the $p_{i,*}$ processes;
    Each process waits to receive the needed segments;
    Each process computes its part of the $D^{(k)}$ matrix;
    }
}



Performance:

Data Size : 1 million (1000 x 1000)

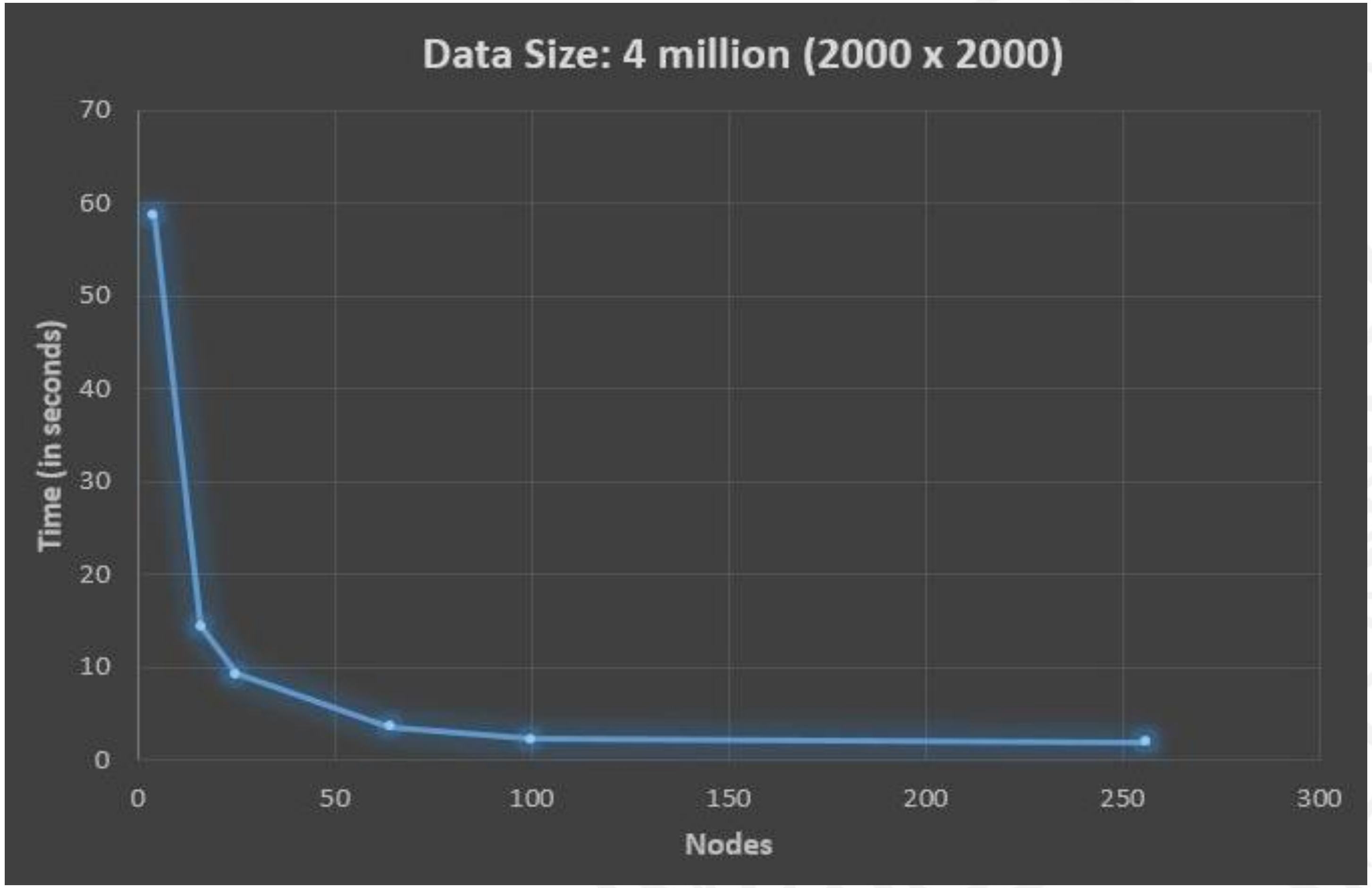
Nodes	Time (in secs)
4	6.587
16	1.646
25	1.075
64	0.519
100	0.3753



Performance:

Data Size : 4 million (2000 x 2000)

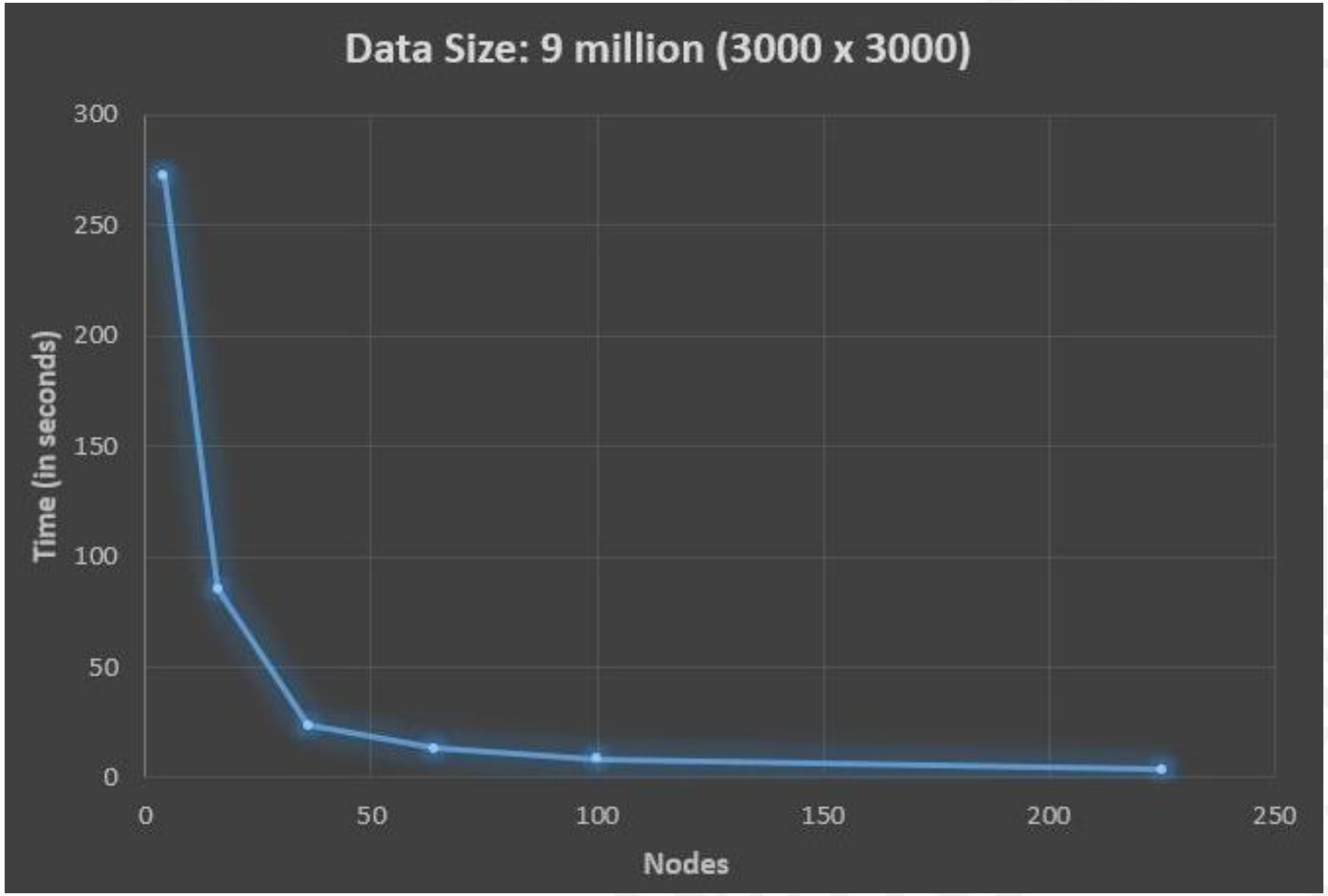
Nodes	Time (in secs)
4	58.654
16	14.452
25	9.332
64	3.627
100	2.35
256	2.01



Performance:

Data Size : 9 million (3000 x 3000)

Nodes	Time (in secs)
4	272.234
16	85.372
36	23.724
64	13.202
100	8.483
225	3.902

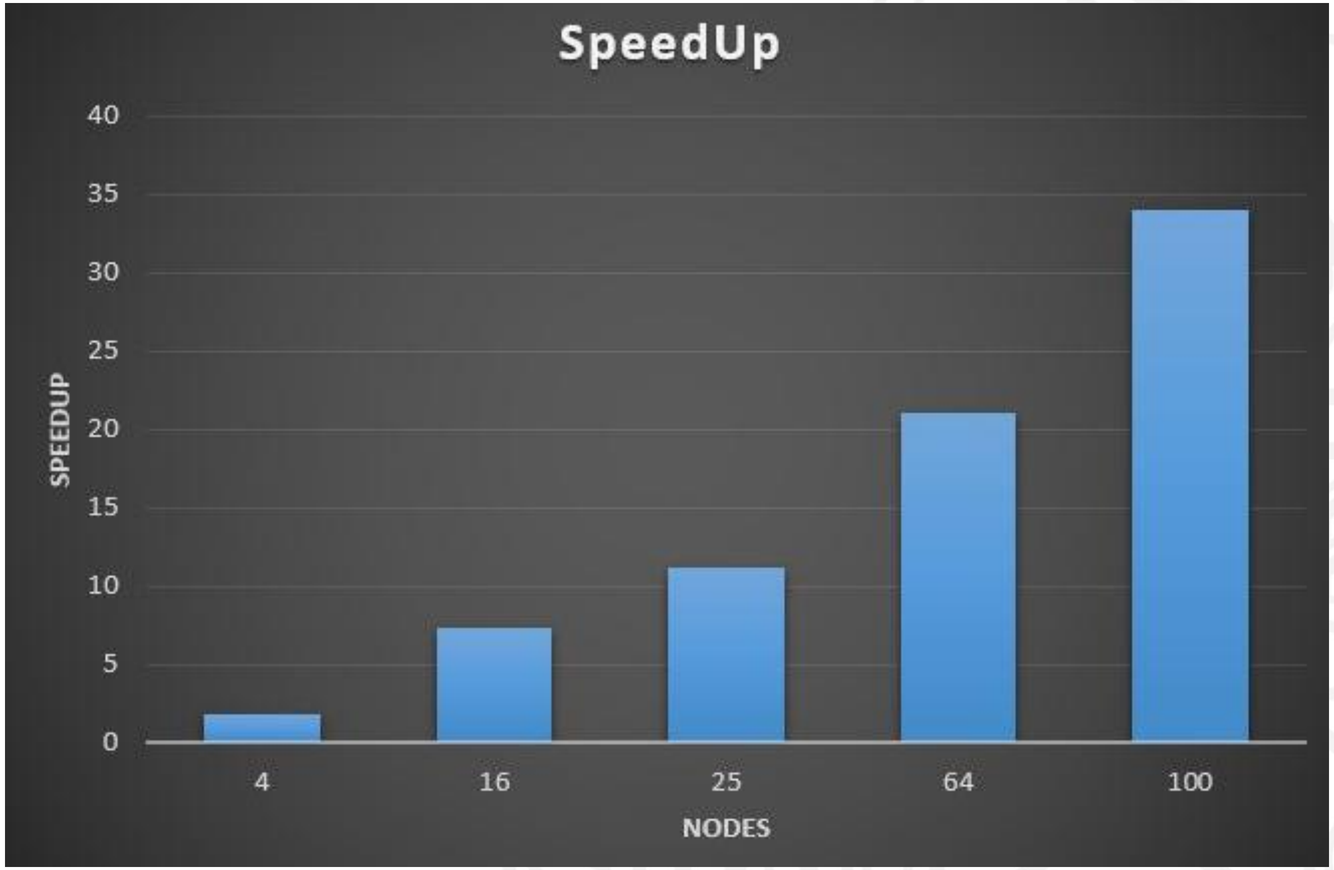


SpeedUp:

Data Size : 1 million (1000 x 1000)

Serial Execution Time: 12 seconds

Nodes	Speed-Up
4	1.821
16	7.29
25	11.162
64	21.089
100	33.99

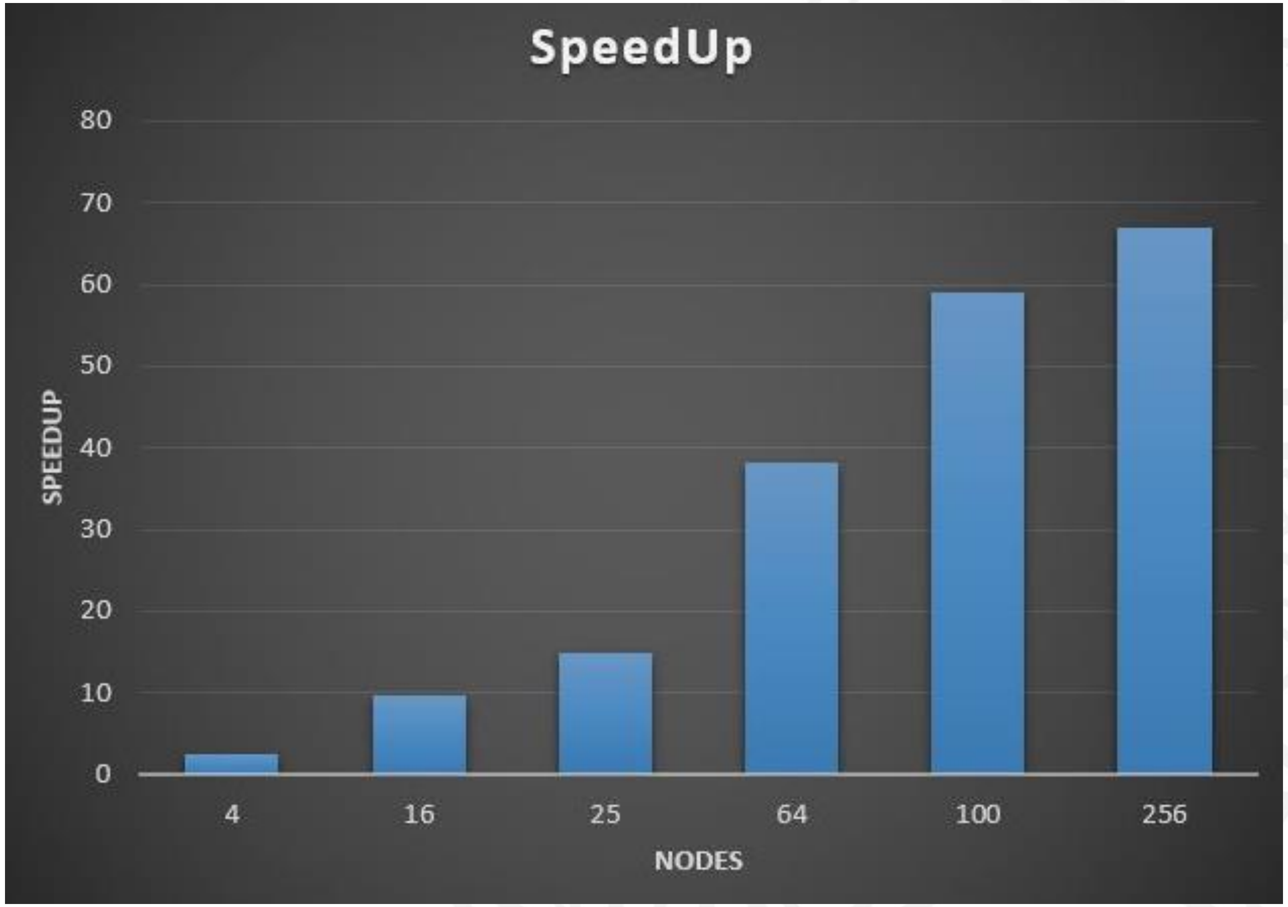


SpeedUp:

Data Size : 4 million (2000 x 2000)

Serial Execution Time: 139 seconds

Nodes	Speed-Up
4	2.369
16	9.61
25	14.89
64	38.3
100	59.14
256	67.05

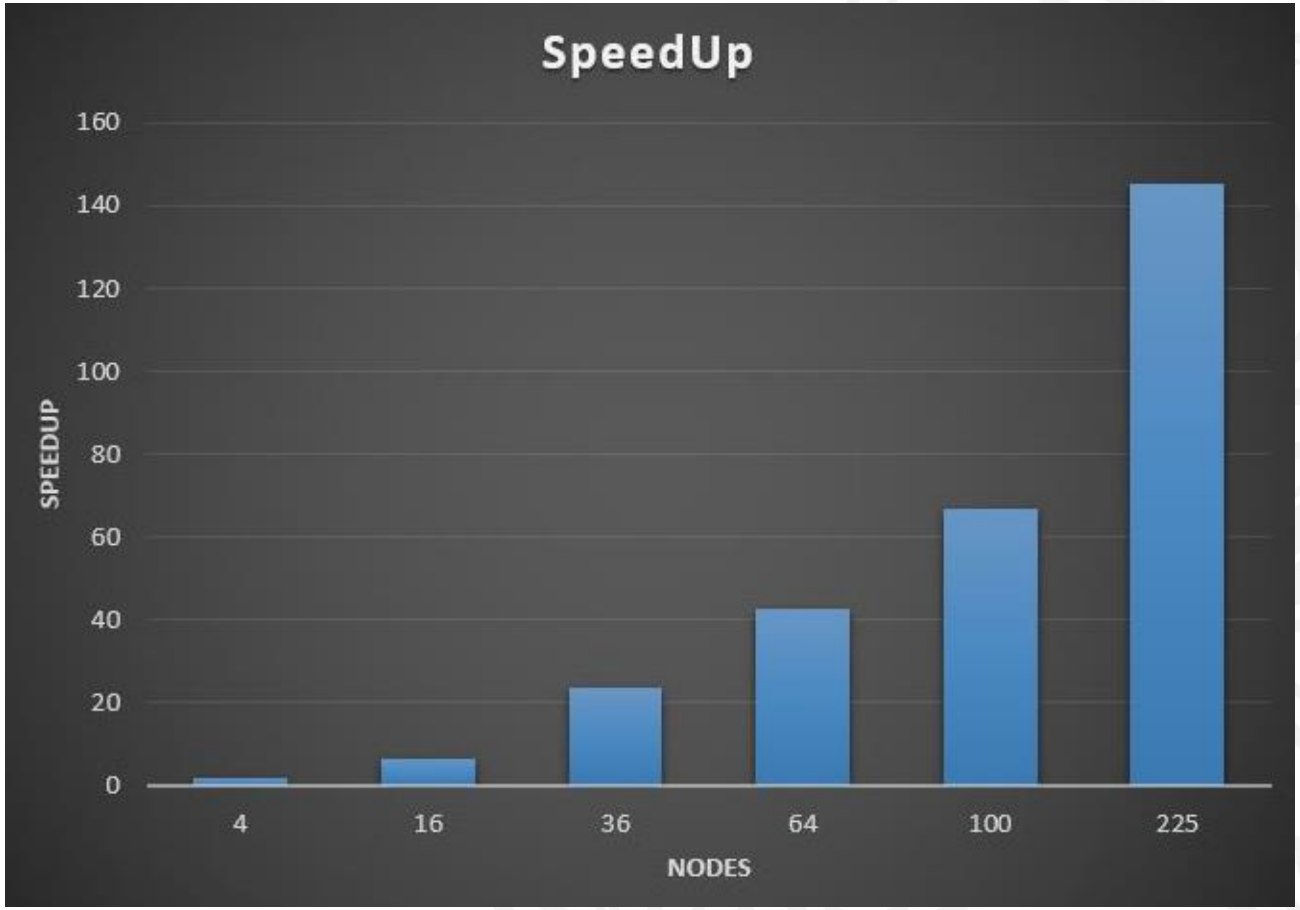


Speed-Up:

Data Size : 9 million (3000 x 3000)

Serial Execution Time: 567 seconds

Nodes	Speed-Up
4	2.082
16	6.641
36	23.899
64	42.94
100	66.839
225	145.3



Challenges:

- Distributing data amongst processors in a 2-D block fashion.
- Communication between respective row and column processors.
- Waiting time for running on 256 (or ~256) nodes.

References:

- https://en.wikipedia.org/wiki/Parallel_all-pairs_shortest_path_algorithm#Parallelization
- http://parallelcomp.uw.hu/ch10lev1sec4.html
- CCR Tutorials and handouts https://ubccr.freshdesk.com/support/solutions/articles/13000026245-tutorials-workshops-and-training-documents

MPI_Bcast("ANY QUESTIONS ???")