Parallelizing LU Decomposition

CSE 633: PARALLEL ALGORITHMS

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Problem Statement

Given a Square matrix A(n x n), decompose it into a Lower triangular matrix (L) and an Upper triangular matrix (U).

A= LU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

Applications

- Computers usually solve system of Linear Equations using LU decomposition
- Used in Computing determinant of matrix

Example: Finding the [U] matrix

- Using the Forward Elimination Procedure of Gauss Elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

Step 1:

$$\frac{64}{25} = 2.56; \quad Row2 - Row1(2.56) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 144 & 12 & 1 \end{bmatrix}$$

$$\frac{144}{25} = 5.76; \quad Row3 - Row1(5.76) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$$

Finding the [U] matrix

- Matrix after Step 1:

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$$

Step 2: $\dfrac{-16.8}{-4.8} = 3.5; \quad Row3 - Row2(3.5) = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$

$$[U] = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Finding the [L] matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}$$

Using the multipliers used during the Forward Elimination Procedure

From the first step of forward elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

$$\ell_{21} = \frac{a_{21}}{a_{11}} = \frac{64}{25} = 2.56$$

$$\ell_{31} = \frac{a_{31}}{a_{11}} = \frac{144}{25} = 5.76$$

Finding the [L] matrix

From the second step of forward elimination

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix}$$

$$\ell_{32} = \frac{a_{32}}{a_{22}} = \frac{-16.8}{-4.8} = 3.5$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix}$$

Parallelizing LU Decomposition

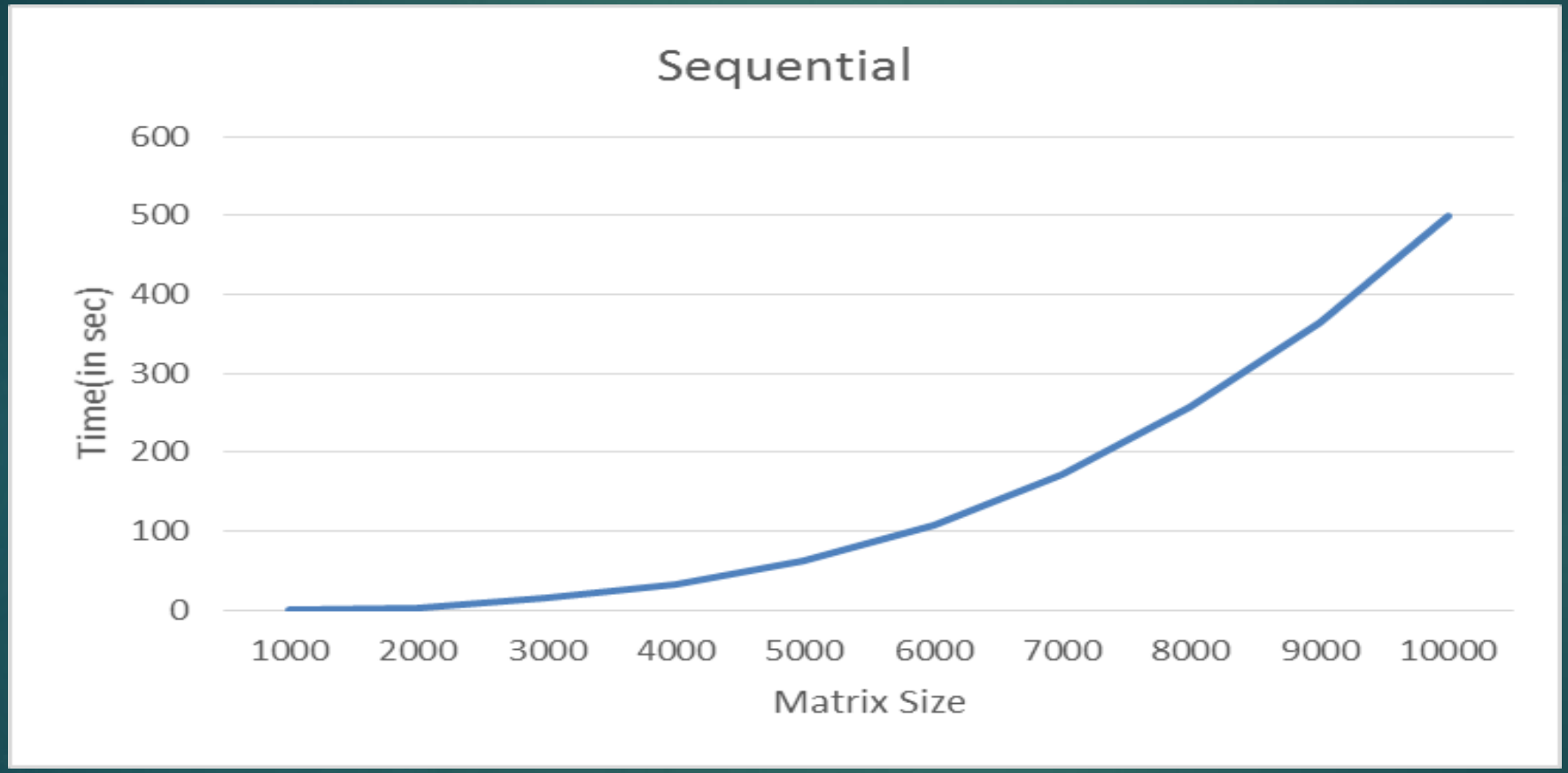
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

- n = number of processors
- Processor rank (0, 1, 2, 3…n-1)
- If (Processor_Rank == (Row_Index % n)):
  - Processes the row
  - Broadcast the row

Implementation

- Sequential
- MPI
- OpenMP

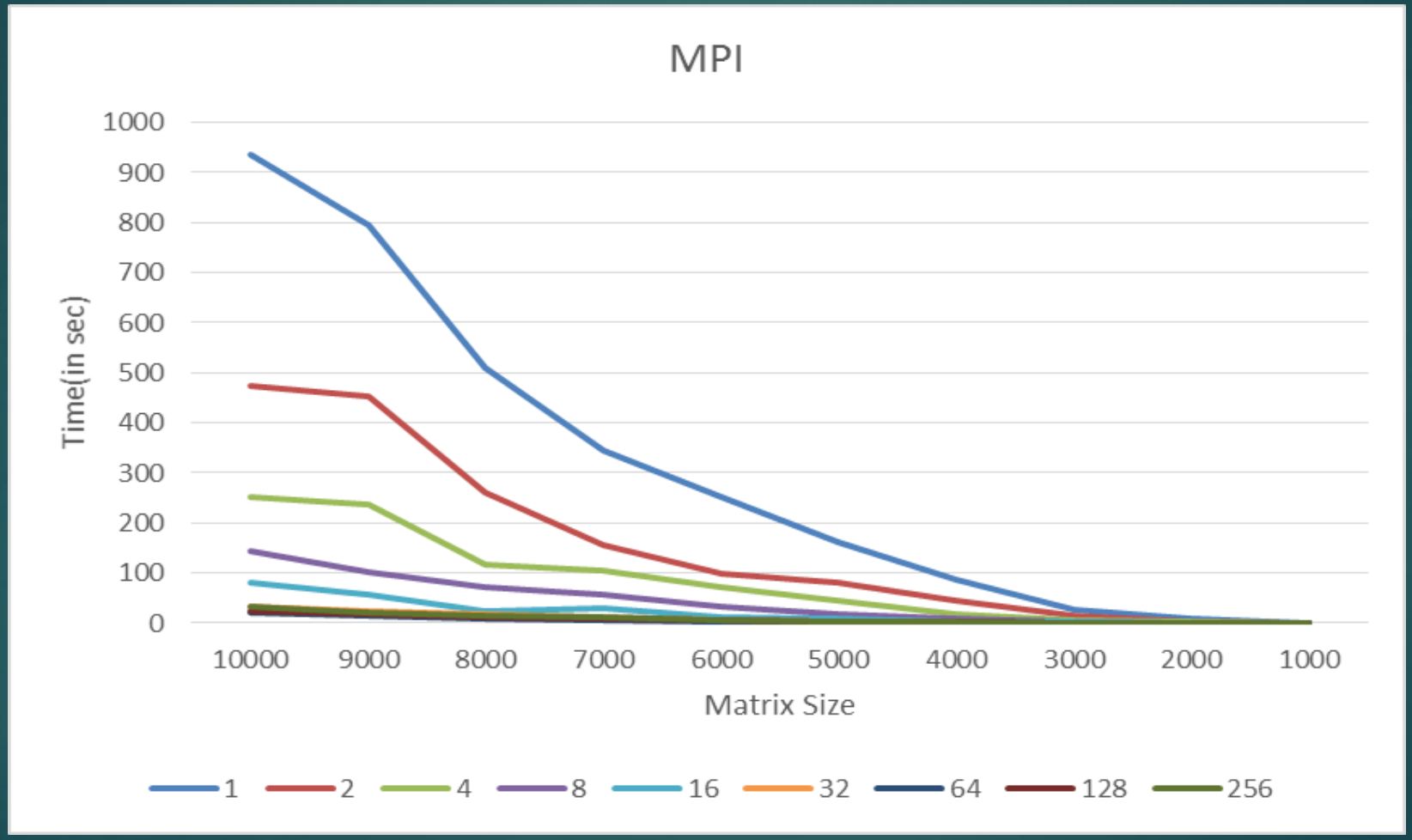
Sequential: Input Size vs Time



MPI: Matrix size vs Nodes

	1	2	4	8	16	32	64	128	256
1000	0.88	0.46	0.25	0.15	0.09	0.06	0.07	0.08	0.1
2000	7.96	3.82	1.88	1.01	0.56	0.35	0.26	0.33	0.4
3000	25.9	13.8	6.7	3.34	1.82	1	0.68	0.8	0.9
4000	87.6	45.7	17.3	8.19	4.07	2.22	1.7	1.92	2.1
5000	162	81.7	45.3	19.1	9.7	4.19	2.66	3.64	4.3
6000	253	99.6	73.2	31.4	10.5	7.12	4.25	5.15	6.77
7000	343	155	105	58.3	28.7	11.8	6.65	9.1	11.4
8000	510	260	118	72.3	25.3	17.3	9.68	12.5	15.7
9000	794	454	237	103	56.5	24.6	14.2	16.4	20.3
10000	937	474	252	143	80.9	34.1	19.5	23.2	32.7

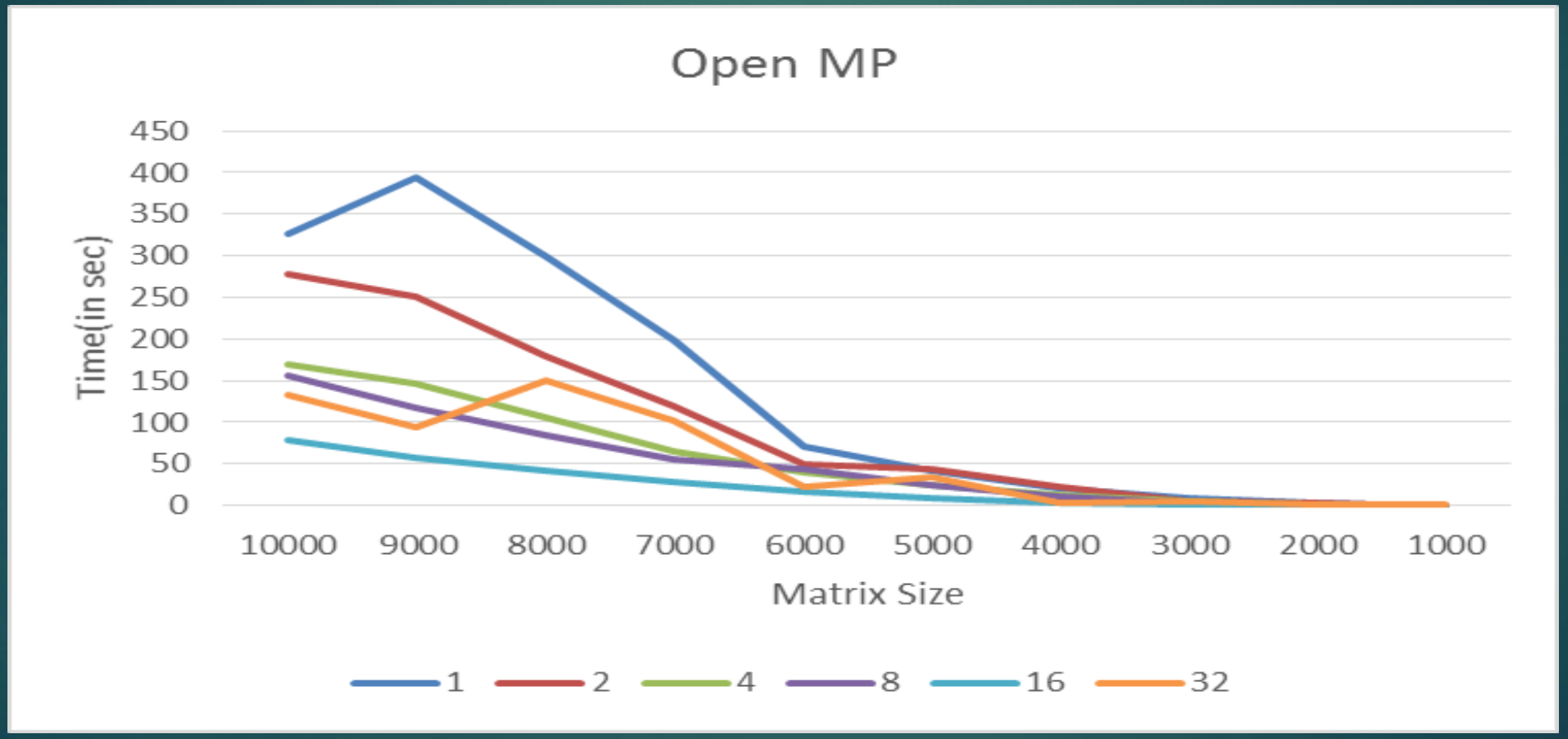
MPI: Input Size vs Time



OpenMP: Matrix size vs Nodes

	1	2	4	8	16	32
1000	0.256	0.16	0.098	0.056	0.28	0.37
2000	2.65	2.36	0.95	0.84	0.57	1.05
3000	9.23	5.99	4.48	3.76	1.23	5.61
4000	21.16	22.21	12.88	10.53	3.29	3.51
5000	41.29	43.89	23.99	25.47	9.05	33.43
6000	71.32	50.28	39.77	43.67	17.6	22.26
7000	198.63	120.2	65.48	54.62	27.5	101.9
8000	299.84	178.62	105.5	84.09	42.8	150.2
9000	393.66	251.16	147.1	118.2	57.4	94.32
10000	325.82	278.54	169.5	156.2	78.6	132.3

OpenMP: Input Size vs Time



References

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