

MST using KRUSKAL's Algorithm

Final presentation

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CSE 708 - Programming Massively
Parallel Systems

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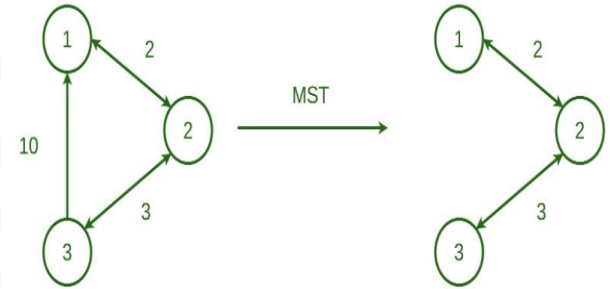
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Minimum Spanning Tree

- A spanning tree is a subset of the edges of the graph that forms a acyclic tree where every node of the graph is a part of the tree.
- MST is
 - a spanning tree
 - Total weight of edges is minimum

Minimum Spanning Tree for Directed Graph



Minimum Spanning Tree

- Number of vertices in graph and MST are same.
- Number of edges = $V-1$ where V is number of vertices
- Need not be unique, multiple MST are possible depending on input.
- Neither disconnected nor cyclic.



Algorithms for MST

- Kruskal's algorithm
- Prim's algorithm
- Boruvka's algorithm



Kruskal's algorithm

1. Sort all the edges in the non-decreasing order of their weights.
2. Select the smallest edge.
3. Check if the selected edge forms a cycle with the MST formed so far
4. Include the edge if no cycle is formed, else discard it.
5. Repeat steps from 2 to 5 till $V-1$ edges are included in the MST.



Approach for Parallelization

- The data's spread across multiple processors.
- Every processor P_i sort the edges that are contained in it's partition V_i - parallelly
- Every processor P_i finds the local MST using the edges in it's partition
 - Some edges are eliminated in this step across all processors



Approach for Parallelization

- Processes merge their local MST's (or MSF's). Merging is performed in the following manner. Let a and b denote two processes which are to merge their local trees (or forests), and let F_a and F_b denote their respective set of local MST edges. Process a sends set F_a to b , which forms a new local MST (or MSF) from $F_a \cup F_b$.
- Merging continues until only one process remains. Its MST is the end result.



Approach for Parallelization

- To create the new local MSF during merge step, we perform Kruskal's algorithm again on $F_a \cup F_b$.
- It can be shown that our approach is efficient for $p = O(n/\log n)$ number of processors.



Communication b/w processors

- Communication between processors happen during the merging the local MSTs into new local MSTs.
- Processor A sends its local MST to Processor B and Processor B calculates the new local MST using A's Local MST and B's local MST.



Implementation in MPI

- I have used MPI to implement the parallel Kruskal Algorithm



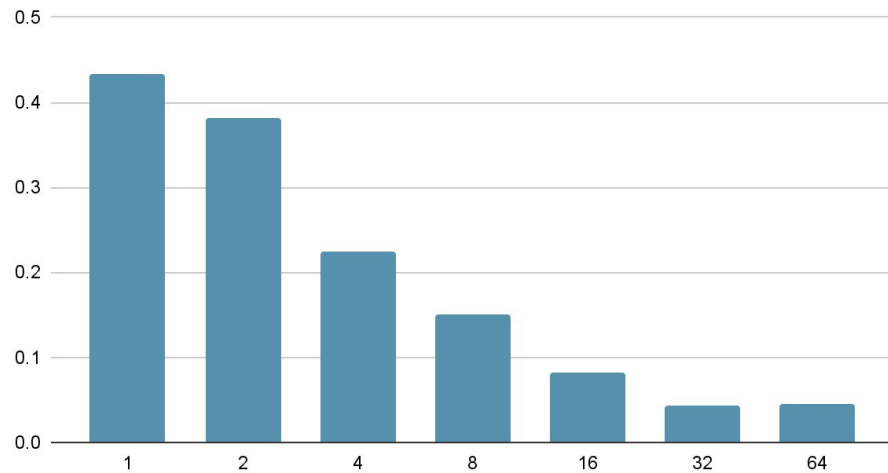
Results



10k vertices - 5% density - 2.5M edges

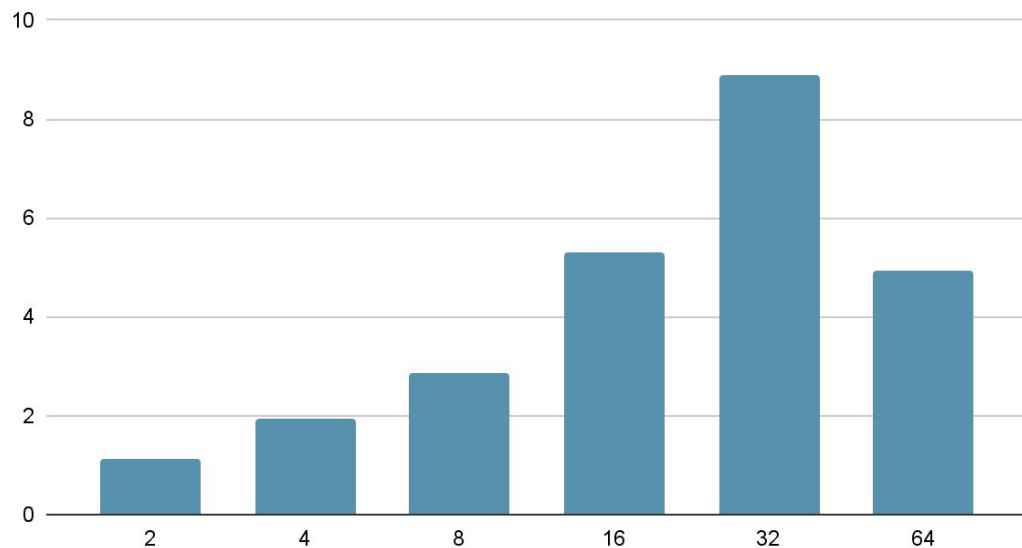
Processors	Time	Speed up
1	0.432887	
2	0.381065	1.135992547
4	0.224477	1.928424738
8	0.150779	2.871003256
16	0.081652	5.301609269
32	0.042923	8.877874333
64	0.045468	4.937032638

Total time vs number of processors



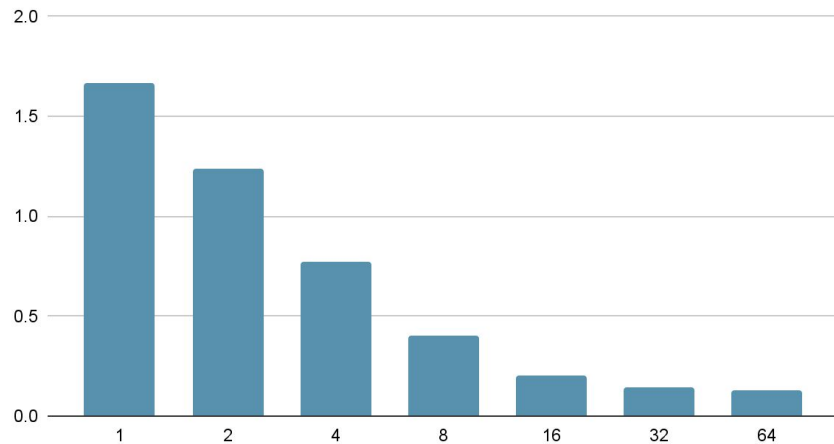
10k vertices - 5% density - 2.5M edges

Speed up graph

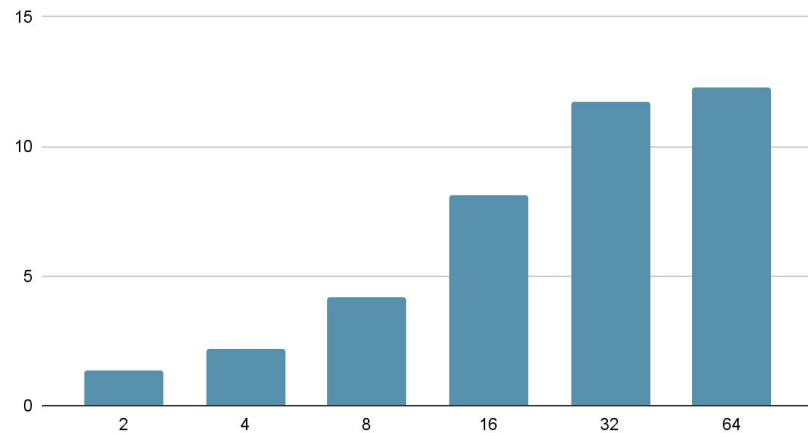


10k vertices - 10 M edges

Total time vs no of processors

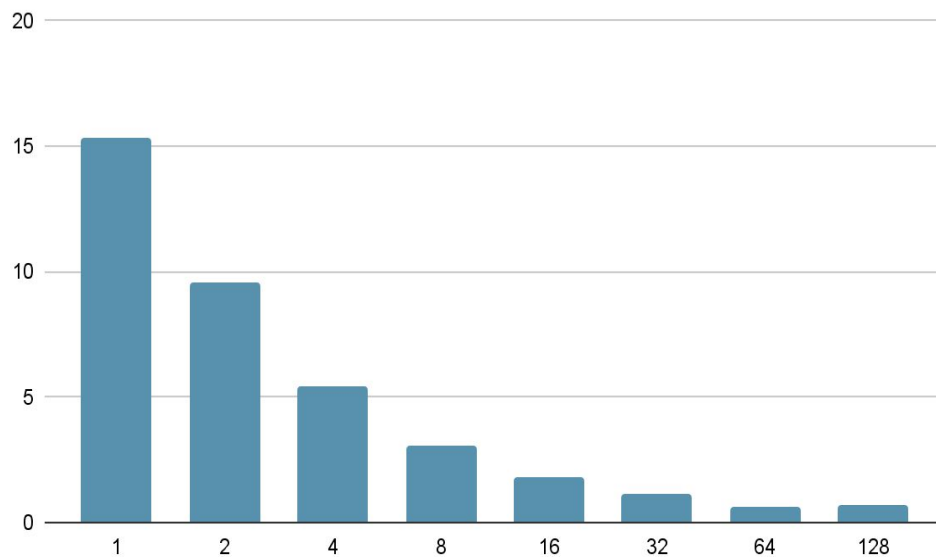


Speed up

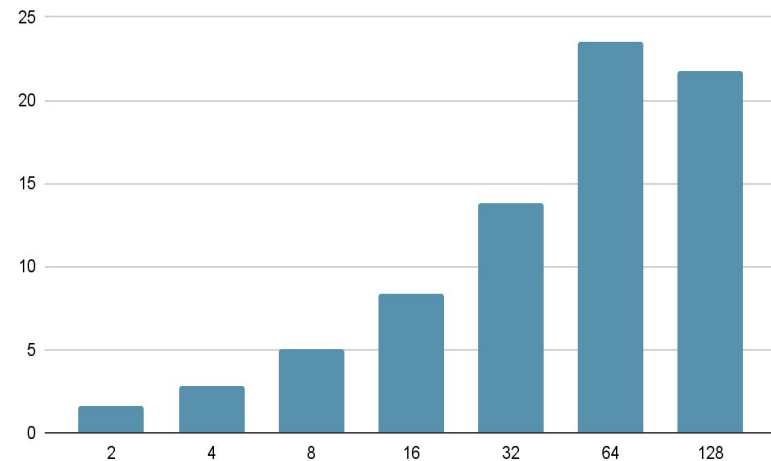


10k vertices - 40M edges

Total time vs no of processors



Speed up



Observations

- Parallel Kruskal performs well on large data.
- The inflection point/ dip is shifting rightwards as we increase the amount of data we are operating with.



References

- [Loncar-TET-Springer.pdf \(scl.rs\)](#)
- [Kruskal's Minimum Spanning Tree \(MST\) Algorithm - GeeksforGeeks](#)



Thank you

