

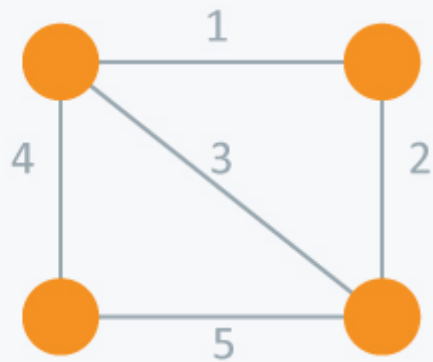
PARALLELIZATION OF PRIM'S ALGORITHM TO FIND THE MST

By Sarath Chandra Reddy Rayapu

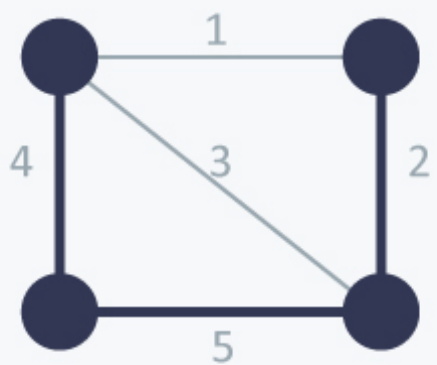


Minimum Spanning Tree (MST) of a graph

- A spanning tree (a tree with all the nodes in the graph) where the sum of the edges is the least possible.

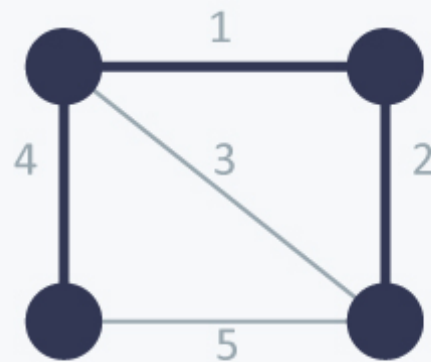


Undirected
Graph



Spanning
Tree

Cost = $11(=4+5+2)$



Minimum Spanning
Tree

Cost = $7(=4+1+2)$



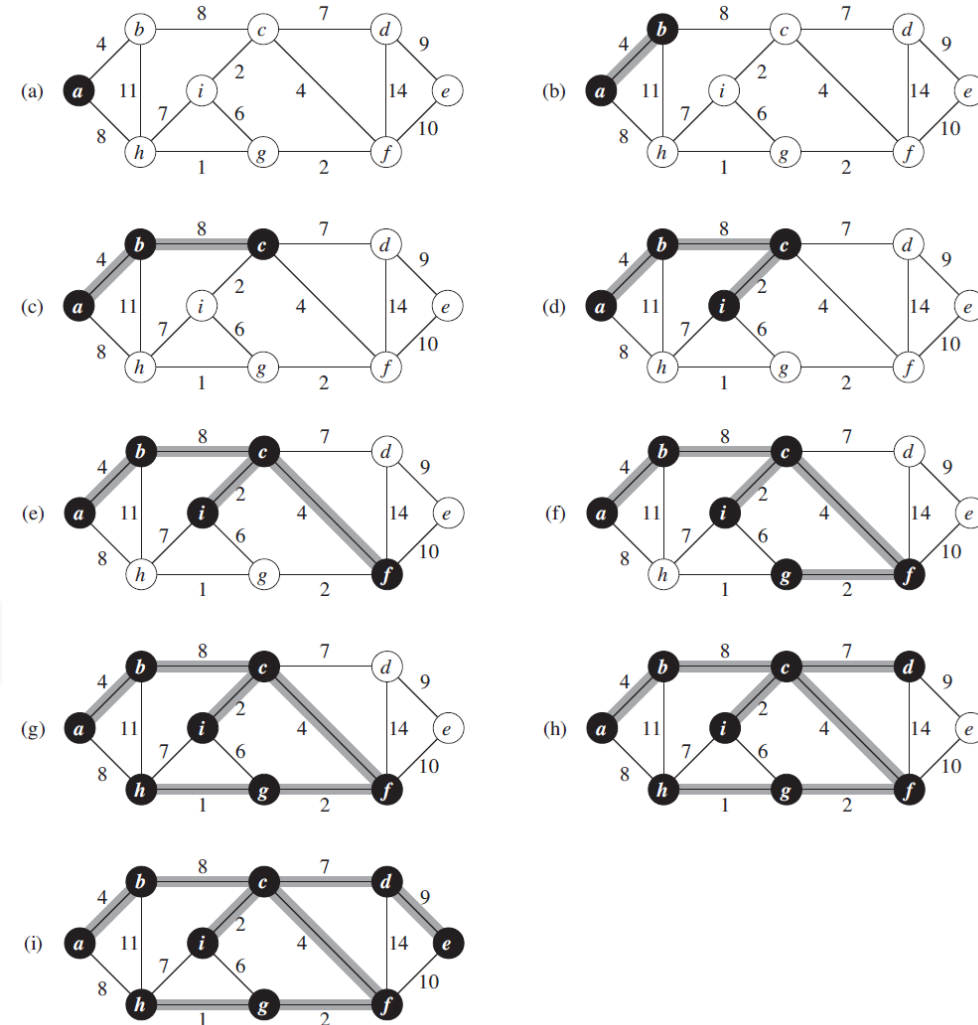
Applications of MST

- Design of cost-effective Networks and efficient Circuits
- Transportation Planning: to determine the most cost-effective routes for building roads, railways, or other transportation networks.
- Image Processing: used in Image Segmentation



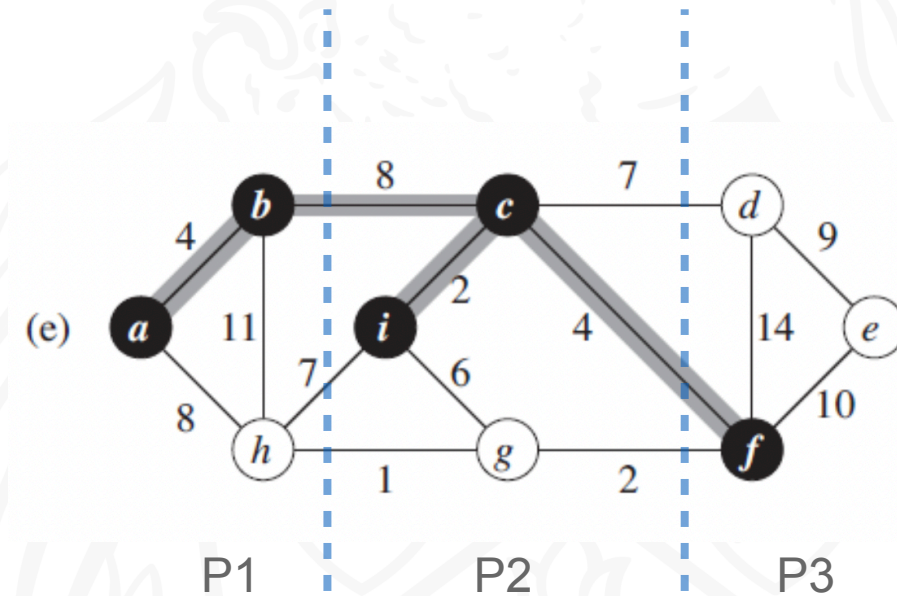
Prim's Algorithm (Sequential):

1. Initialize a tree with a single vertex, chosen arbitrarily from the graph.
2. Grow the tree by one edge: Of the edges that connect the tree to vertices not yet in the tree, find the minimum-weight edge, and transfer it to the tree.
3. Repeat step 2 (until all vertices are in the tree)
4. Time = $O(n^2)$

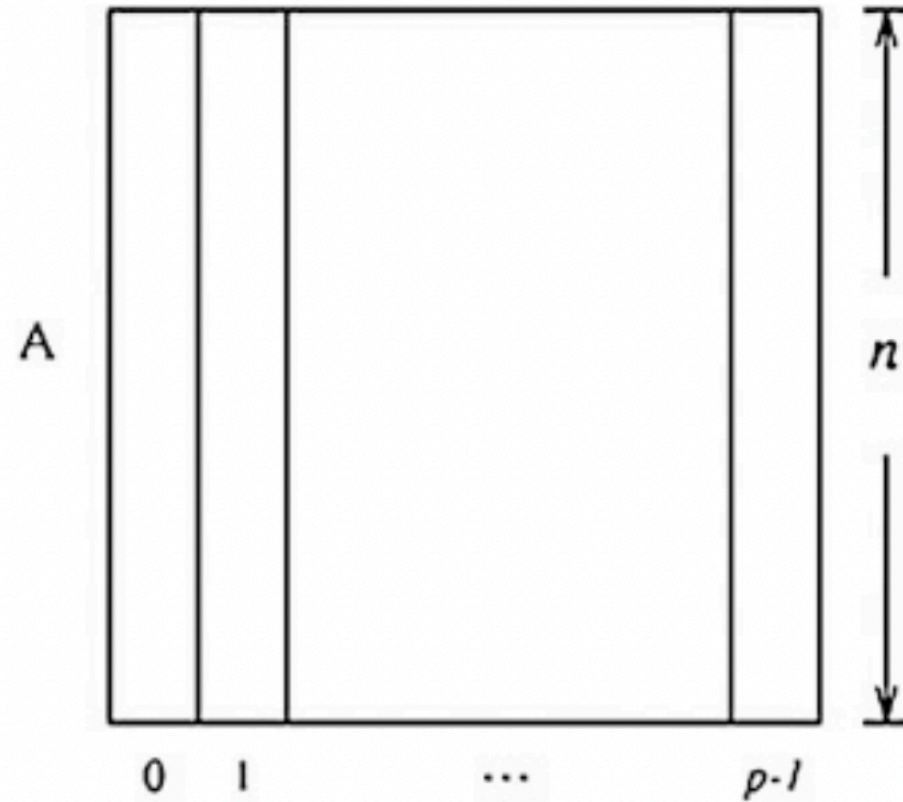


Pseudo code for Parallel approach

- Initialization:
- Divide the set of vertices V into p subsets V_1, V_2, \dots, V_p
- Assign each subset to a different process
- While vertices_in_MST is not equal to V :
- For each process p_i :
 - Find the minimum-weight edge e_i (candidate) connecting MST to vertices in V_i
 - Send e_i to the root process using MPI_Reduce to find the global minimum-weight edge e_{min}
- If rank of current process is root:
 - Select the minimum-weight edge e_{min} from the received edges
 - Add e_{min} to MST
- Broadcast e_{min} to all processes
- Continue this till all the vertices are in the MST
- Time = $O(n^2/p) + O(n \log p)$

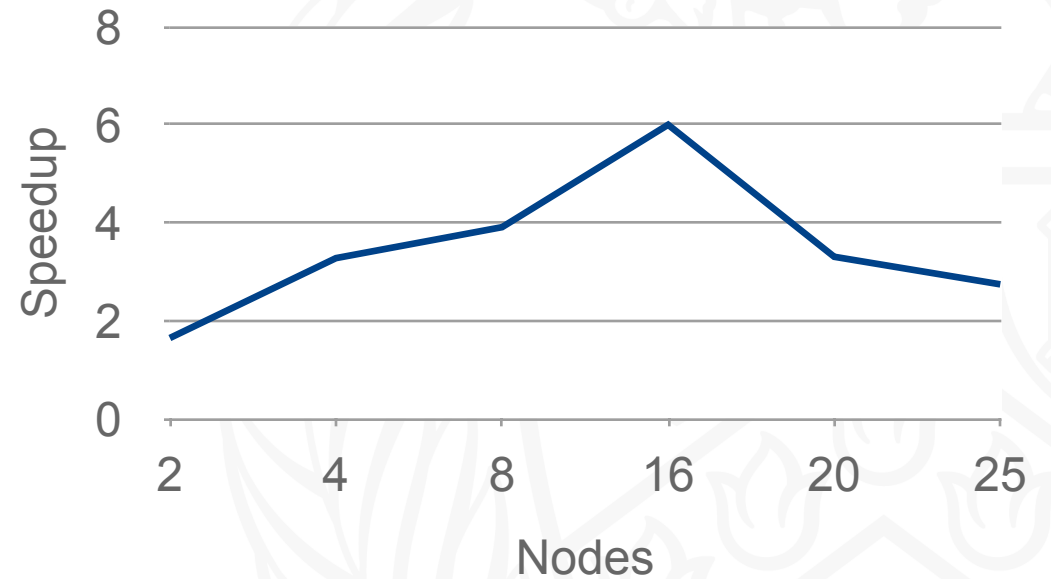
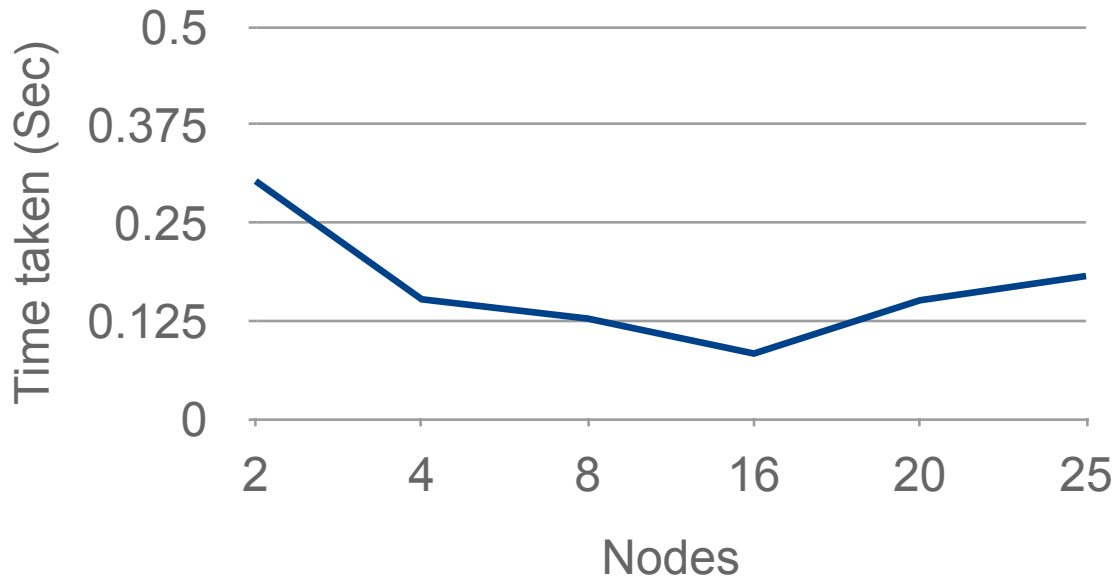


- Partitioning of adjacency matrix among 'p' processors:



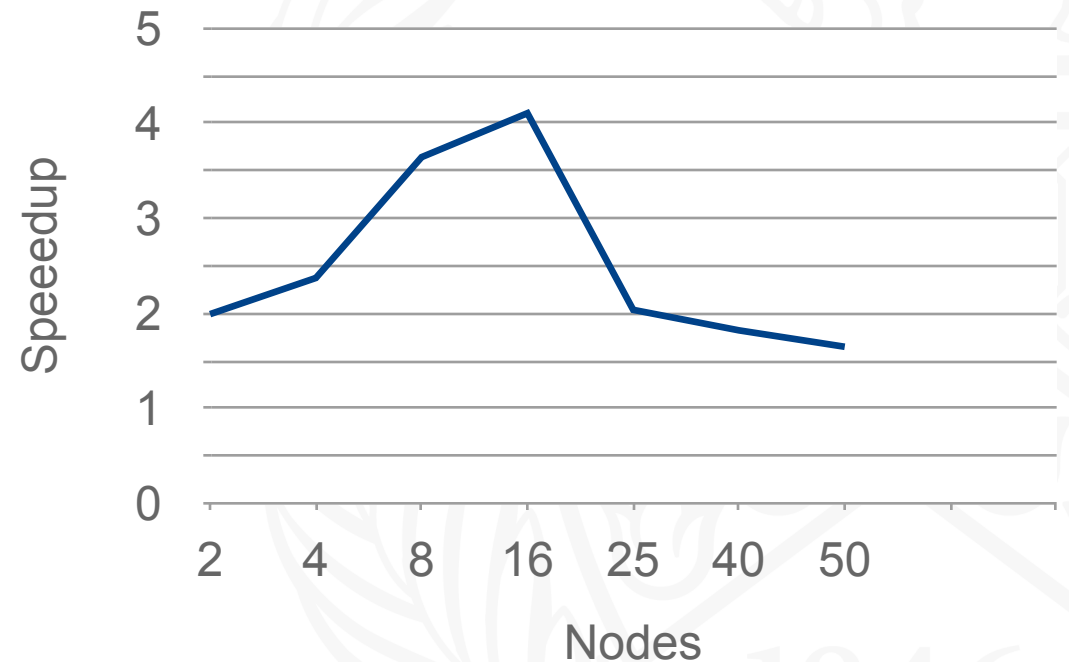
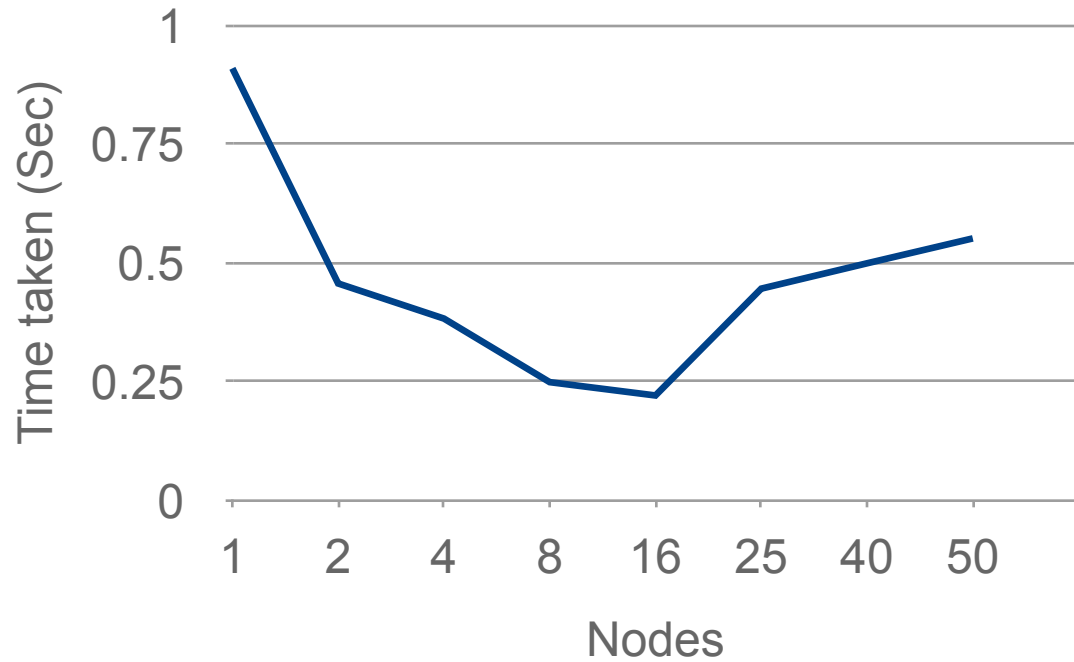
Results

- Input graph: 10000 nodes (5% density)



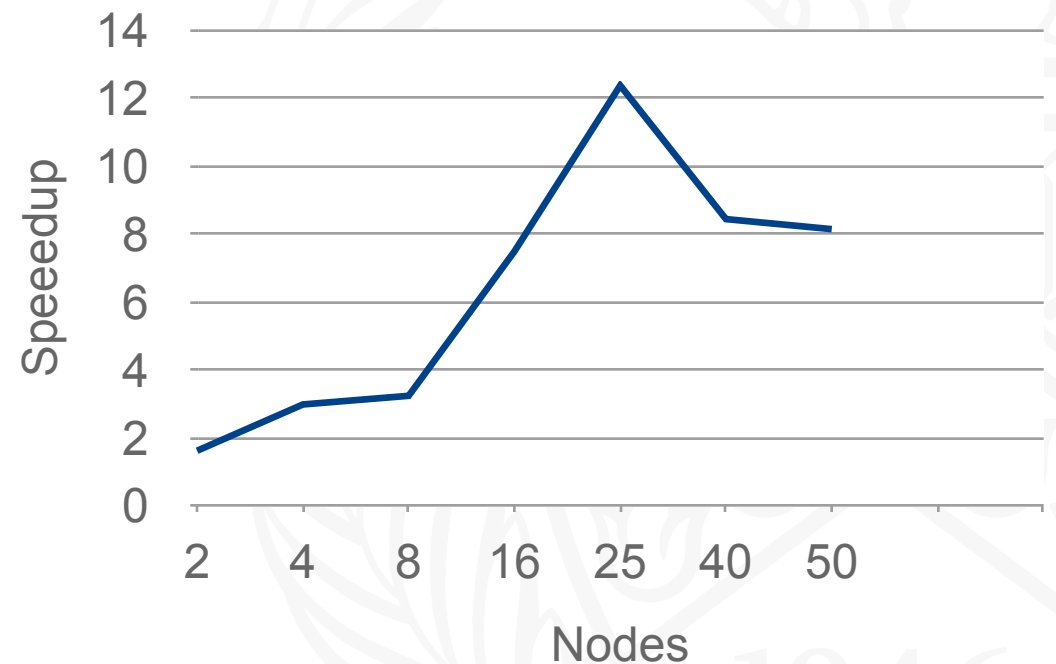
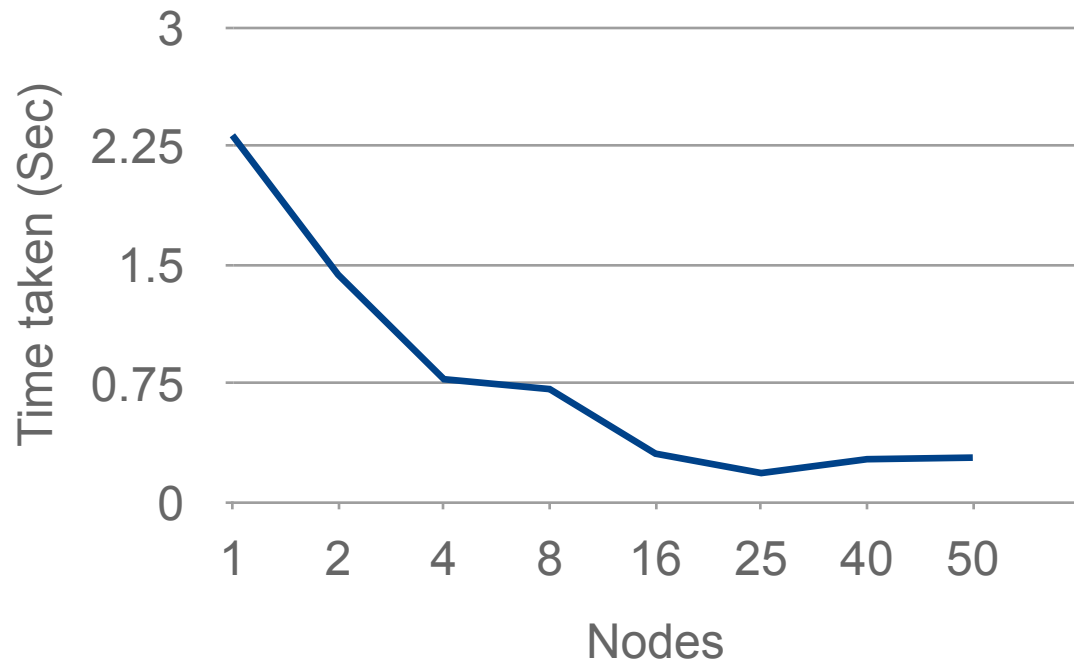
Results

- Input graph: 10000 nodes (10% density)



Results

- Input graph: 10000 nodes (20% density)



Observations

- This algorithm works best with larger datasets by gaining considerable speedups.
- Also, higher density graphs are better suited for this as we are using an adjacency matrix to store the graph.



References

- Parallelization of Minimum Spanning Tree Algorithms Using Distributed Memory Architectures
<http://www.scl.rs/papers/Loncar-TET-Springer.pdf>



THANK YOU

