Elements of a new constructional system

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Abstract

For my parents

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Chapter 1

A new tool for an old project

1.1 The old project

The general aim of the present dissertation is the same as the general aim of Rudolf Carnap's *Aufbau*: the outline of a constructional system, i.e., the sketch of a conceptual system which undertakes not only "the division of concepts into various kinds and the investigation of the differences and mutual relations between these kinds" but also "a step-by-step derivation or 'construction' of all concepts from certain fundamental concepts, so that a genealogy of concepts results in which each one has its definite place".[1] Nonetheless, my project is not exactly the same as Carnap's. Three notable specific differences between them counterbalance two notable specific similarities.

The first notable specific similarity lies in the fact that both projects rely on the use of an all-encompassing formal system. Needless to say, this makes me an advocate of an unpopular philosophical style. Even if one's attention is focused on contemporary analytic philosophy, where most of the living philosophers who value exactness of thought are to be found, one cannot help noticing that formal philosophy is often treated with suspicion.

[1] [9], page 5.

Most analytic philosophers today agree with Hao Wang that, typically, the use of formal systems in philosophy is comparable to “the use of an airplane to visit a friend living in the same town”, or to “using a huge computer solely to calculate the result of multiplying seven by eleven”.[2] Besides, the few pieces of formal philosophy which have reached the limelight are almost always downplayed as, at best, a vindication of the occasional legitimacy of applying a variety of formal systems in an *ad hoc* manner. Nowadays, if a philosopher endows a formal system with the honourable status of a step towards Bertrand Russell’s ideal of a logically perfect system—the ideal of a system “in which everything that we might wish to say in the way of propositions that are intelligible to us, could be said, and in which, further, structure would always be made explicit”[3]—he risks penalties ranging from his peers’ polite puzzlement to their relentless contempt. Some eminent living philosophers have built their reputations on their studies of the *Aufbau*, but the nature of these studies is more exegetical than epistemological. Carnap’s project, whose plausibility depends on the plausibility of Russell’s ideal, has been relegated to the heap of glorious failures, where it sits not far away from Scott’s second Antarctic expedition. The odds against the recognition of my project are thus nothing short of formidable.

So why have I chosen to pursue it? Am I not aware that I may end up restating platitudes in pedantic obscurity, or worse, working on problems which are mere by-products of a particular use of a particular formal system? Well, I offer no apologies. Although studying the application of Hintikka’s constituents to the problem of truthlikeness has made me duly aware of these dangers, my call to avoid professional sins of omission remains stronger than my call to avoid professional sins of commission. It is my firm opinion that an extravagant attempt to solve a philosophical problem is preferable to no attempt at all. Of course, one could accept this view and yet deny the existence or manageability of an all-encompassing formal system. Wolfgang Stegmüller, in the process of promoting a constructional project

[2][44], page 233.

[3][37], page 123.

which relies on the use of informal set theory, presents this criticism:

> One need not be a convinced follower of Quine's scepticism with regard to possible worlds in order to differentiate between logical possibilities and *realistic* possibilities. The reconstruction of physical theories within the framework of the [Carnap approach] will, for a very, very long time, be only a logical possibility but not a realistic one. Therefore, adherents of [the Carnap approach] are forced to use simple, fictitions [*sic*] examples instead of instances from real science.
>
> Let us illustrate this with an example from mathematics. . . . suppose that Bourbaki had made up his mind to use, as a universal language of mathematics, *formalized set theory* instead of *informal set theory* for his intended reconstruction of modern mathematics in precise terms. The Bourbaki volumes would then look like, for example, the last chapter of Shoenfield's *Mathematical Logic*. I really fear that the Bourbaki volumes would not exist, at least not yet, since instead of having published more than 20 volumes, Bourbaki would still be working on his first volume.
>
> By rejecting the Carnap approach I seem to be joining those who have always contended that Carnap's main mistake consisted in overestimating the power of modern logic. But this is, of course, nonsense. It was one of the great and incontestable philosophical merits of Carnap that he recognized very early the importance of modern logic as an irreplaceable tool for many analytic purposes. If he misjudged something, it was not modern logic but our *human abilities* to handle this powerful tool.[4]

I have two objections to it. Firstly, we need not consider the Bourbaki programme and the project with which Stegmüller compares it as competitors. We can say that the former is to

[4] [40], page 5.

the latter what the compound light microscope is to the transmission electron microscope. Therefore, we can also say that, being subject to quite different but equally legitimate evaluation criteria, Carnap's project and Stegmüller's could live in harmony with each other. Secondly, even though the amount of co-operation required for the advancement of Carnap's project exceeds the impressive amount of co-operation exhibited by Bourbaki, we should not deem it a mere logical possibility. Philosophy has always been an odd assortment of warring petty clans, but this is not a manifestation of a law of nature. If philosophers and computer scientists belong to the same species, then, with the right incentives and support, it is realistically possible to persuade a large number of philosophers to work together in order to advance a project as ambitious as Carnap's. I hope that mine does not prove to be entirely unworthy of these two lines of defence.

The second notable specific similarity lies in the fact that both projects have a claim to epistemological relevance. I intend to follow in Carnap's footsteps and present the reader with an intelligible suggestion as to how scientific and everyday concepts are to be reduced to the "given". That is to say, I share Carnap's intent to adumbrate the "logical progress which leads, first, to the various entities of my consciousness, then to the physical objects, furthermore, with the aid of the latter, to the phenomena of consciousness of other subjects, i.e., to the heteropsychological, and, through the mediation of the heteropsychological, to the cultural objects".[5] Consequently, all physical entities fall outside the basis of my constructional system, including the elementary particles and the points of the four-dimensional spacetime continuum.

The first notable specific difference lies in the fact that only Carnap's project is hostile to metaphysics. The *Aufbau* is an exercise both in construction and in demolition, for Carnap sees the existence of a formal system in which all scientific and everyday concepts but no metaphysical concepts are constructible as proof that metaphysics is an ill-formed discipline. Although I would not go as far as agreeing with William Kneale that "posi-

[5] [9], page 286.

tivism of that kind is related to dogmatic metaphysics in much the same way as fascism is related to communism", I most definitely do not buy it.[6] Philosophy is for me what it is for Hector-Neri Castañeda: not only "the scrutiny of the general, constitutive patterns of the different types of experience" but also "the search for the most general and pervasive structural aspects of *reality* [emphasis added]".[7] Besides, I doubt that one can play one of these games without playing the other. Since Carnap's project relies on the use of a formal system with variables ranging over classes of classes of concreta, a hard-core nominalist will never accept it,[8] and so Carnap's view of it as the neutral foundation which all prominent metaphysical schools have in common is unsustainable.

The second notable specific difference lies in the fact that only Carnap's project rests on the thesis of extensionality, i.e., on the assumption that, "in every statement about a concept, this concept may be taken extensionally".[9] The thesis of extensionality made sound philosophical sense in a time when the view of logic as a theory of membership reigned supreme, but, quite rightly, this consensus has eroded since Richard Montague's exhortation to use set theory to "'justify' a language or theory that transcends set theory, and then proceed to transact a new branch of philosophy within the new language".[10] However harmless it may look in the *Aufbau*, the thesis of extensionality prevents the philosopher of science from properly constructing what matters the most to him: empirical theories. (It forces him to identify any given empirical theory either with the True or with the False or with nothing at all.) My project rests on the contrary assumption that a concept must be taken intensionally in every statement about it, which strikes me as a much better starting point for the construction of these crucial entities.

The third notable specific difference lies in the fact that only my project completely dispenses with concreta. Naturally, there is nothing wrong in principle with countenancing

[6] [23], page 342.
[7] [11], page 45.
[8] For an elaboration of this point, see section 2 of "A world of individuals", in [4].
[9] [9], page 72.
[10] [26], page 155.

concreta, but the concreta which Carnap sanctions, the temporal cross sections of a person's stream of experience, have a serious defect: we find ourselves in deep philosophical trouble whenever we try to locate them. Where is the temporal cross section of my stream of experience of which my first visual image of Stockholm is a part? All of the stock answers of contemporary analytic philosophy—that it is in Stockholm itself, that it is literally inside my head, that it is in a spatial world of its own—can be easily refuted. This raises the question of whether or not one can devise an epistemologically relevant constructional system without engaging in talk of sense data. Naive realism, which advises us to take medium-sized physical bodies as basic elements, is not a plausible reply to the argument from illusion, and so the option of doing away with concreta, which Carnap does not consider, might turn out to be rewarding. What I propose is to countenance nothing but senses. That is to say, my project rests on the seemingly outlandish hypothesis that these entities are all that we are ever acquainted with.

1.2 The new tool

My formal system, which I shall call "**KFGS5a**", is a slight variant of **KFGS5**, Raymond Turner's artful combination of the untyped lambda calculus, the Kripke-Feferman-Gilmore theory of truth and **S5**.[11]

The grammar of **KFGS5a** is as follows:

- Alphabet
  - Variables $x_1, x_2, x_3, \ldots$
  - Individual constants $c_1, c_2, c_3, \ldots$
  - Predicates $=, T, F, P, A_1, A_2, A_3, \ldots$
  - Sentential connectives $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$

[11] For Turner's presentation of **KFGS5**, see [42], chapter 3 and section 7.1.

- Modal connectives $\Box, \Diamond$
- Variable-binding operators $\lambda, \exists, \forall$
- Other symbols $., [,], (,)$

- Terms and formulas
  - Any variable is a term.
  - Any individual constant is a term.
  - If τ is a term, and α is a variable, then $[\lambda\alpha.\tau]$ is a term.
  - If τ and υ are terms, then $[\tau\upsilon]$ is a term.
  - If φ is a formula, then $[\varphi]$ is a term.
  - If τ and υ are terms, then, for each $i \geq 1$, $\tau = \upsilon$, $T\tau$, $F\tau$, $P\tau$ and $A_i\tau$ are formulas.
  - If φ and ψ are formulas, then $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, $\neg\varphi$, $(\varphi \rightarrow \psi)$, $(\varphi \leftrightarrow \psi)$, $\Box\varphi$ and $\Diamond\varphi$ are formulas.
  - If φ is a formula, and α is a variable, then $(\exists\alpha)\varphi$ and $(\forall\alpha)\varphi$ are formulas.
  - Nothing is a term or formula except by virtue of the foregoing eight rules.

An occurrence of a variable α is bound in a term or formula φ just in case it stands within an occurrence in φ of an expression $(\exists\alpha)\psi$, $(\forall\alpha)\psi$ or $[\lambda\alpha.\psi]$, where ψ is a term or formula. An occurrence of a variable is free in a term or formula φ just in case it stands within φ but is not bound in φ. A variable itself is bound or free in a term or formula φ according as there is a bound or free occurrence of it in φ.

For any term or formula φ, term τ and variable β, the expression obtained from φ by properly substituting τ for β—use "$s(\varphi, \tau, \beta)$" to abbreviate this—is defined as follows:

- If φ is a variable, this holds: if φ is β, $s(\varphi, \tau, \beta)$ is τ; if φ is not β, $s(\varphi, \tau, \beta)$ is φ.

- If φ is an individual constant, $s(\varphi, \tau, \beta)$ is φ.

- If φ is an expression $[\lambda\alpha.\upsilon]$, where α is a variable and υ is a term, this holds: if β is not free in φ, $s(\varphi, \tau, \beta)$ is φ; if β is free in φ and α is not free in τ, $s(\varphi, \tau, \beta)$ is

$$[\lambda\alpha.s(\upsilon, \tau, \beta)]\ ;$$

  if β is free in φ and α is free in τ, $s(\varphi, \tau, \beta)$ is

$$[\lambda\chi.s(s(\upsilon, \chi, \alpha), \tau, \beta)]\ ,$$

  where χ is the first variable in the list $x_1, x_2, x_3, \ldots$ which is not free in τ.

- If φ is an expression $[\upsilon\upsilon\prime]$, where υ and $\upsilon\prime$ are terms, $s(\varphi, \tau, \beta)$ is

$$[s(\upsilon, \tau, \beta)s(\upsilon\prime, \tau, \beta)]\ .$$

- If φ is an expression $[\psi]$, where ψ is a formula, $s(\varphi, \tau, \beta)$ is

$$[s(\psi, \tau, \beta)]\ .$$

- If φ is an expression $\upsilon = \upsilon\prime$, where υ and $\upsilon\prime$ are terms, $s(\varphi, \tau, \beta)$ is

$$s(\upsilon, \tau, \beta) = s(\upsilon\prime, \tau, \beta)\ .$$

- If φ is an expression $\xi\upsilon$, where υ is a term and, for some $i \geq 1$, ξ is T, F, P or A_i, $s(\varphi, \tau, \beta)$ is

$$\xi s(\upsilon, \tau, \beta)\ .$$

- If φ is an expression $\xi\psi$, where ψ is a formula and ξ is $\neg$, $\Box$ or $\Diamond$, $s(\varphi, \tau, \beta)$ is

$$\xi s(\psi, \tau, \beta) \ .$$

- If φ is an expression $(\psi\xi\psi\prime)$, where ψ and $\psi\prime$ are formulas and ξ is $\wedge$, $\vee$, $\rightarrow$ or $\leftrightarrow$, $s(\varphi, \tau, \beta)$ is

$$(s(\psi, \tau, \beta)\xi s(\psi\prime, \tau, \beta)) \ .$$

- If φ is an expression $(\xi\alpha)\psi$, where α is a variable, ψ is a formula and ξ is $\exists$ or $\forall$, this holds: if β is not free in φ, $s(\varphi, \tau, \beta)$ is φ; if β is free in φ and α is not free in τ, $s(\varphi, \tau, \beta)$ is

$$(\xi\alpha)s(\psi, \tau, \beta) \ ;$$

  if β is free in φ and α is free in τ, $s(\varphi, \tau, \beta)$ is

$$(\xi\chi)s(s(\psi, \chi, \alpha), \tau, \beta) \ ,$$

  where χ is the first variable in the list $x_1, x_2, x_3, \ldots$ which is not free in τ.

The passage from a term or formula to an expression of English, which will frequently be indispensable, must proceed on the basis of a scheme of abbreviation. Let us understand by an abbreviation an ordered pair of expressions, the first of which is an individual constant and the second an English name or nominal expression. A scheme of abbreviation is a collection of abbreviations such that no two abbreviations in the collection have the same first member. The process of literal translation into English on the basis of a given scheme of abbreviation begins with a term or formula and ends with an expression of English. The process consists of the following steps:

- Replace individual constants by English names or nominal expressions in accordance with the given scheme of abbreviation.

- Replace all occurrences of $[\tau\upsilon]$, where τ and υ are terms, by $\ulcorner$[the outcome of the application of τ to υ]$\urcorner$.
- Replace all occurrences of $[\lambda\alpha.\tau]$, where α is a variable and τ is a term, by $\ulcorner$[the command to output τ, for any sense α as input]$\urcorner$.
- Replace all occurrences of $[\varphi]$, where φ is a formula, by $\ulcorner$[the thought that φ]$\urcorner$.
- Replace all occurrences of $\tau = \upsilon$, where τ and υ are terms, by $\ulcorner\tau$ is identical to $\upsilon\urcorner$.
- Replace all occurrences of $T\tau$, where τ is a term, by $\ulcorner\tau$ is true$\urcorner$.
- Replace all occurrences of $F\tau$, where τ is a term, by $\ulcorner\tau$ is false$\urcorner$.
- Replace all occurrences of $P\tau$, where τ is a term, by $\ulcorner\tau$ is a proposition$\urcorner$.
- For each $i \geq 1$, replace all occurrences of $A_i\tau$, where τ is a term, by $\ulcorner\tau$ is a\an i-place attribute$\urcorner$.
- Replace all occurrences of $(\varphi \wedge \psi)$, where φ and ψ are formulas, by $\ulcorner(\varphi$ and $\psi)\urcorner$.
- Replace all occurrences of $(\varphi \vee \psi)$, where φ and ψ are formulas, by $\ulcorner(\varphi$ or $\psi)\urcorner$.
- Replace all occurrences of $\neg\varphi$, where φ is a formula, by $\ulcorner$it is not the case that $\varphi\urcorner$.
- Replace all occurrences of $(\varphi \rightarrow \psi)$, where φ and ψ are formulas, by $\ulcorner$(if φ, then $\psi)\urcorner$.
- Replace all occurrences of $(\varphi \leftrightarrow \psi)$, where φ and ψ are formulas, by $\ulcorner(\varphi$ if and only if $\psi)\urcorner$.
- Replace all occurrences of $\Box\varphi$, where φ is a formula, by $\ulcorner$it is necessary that $\varphi\urcorner$.
- Replace all occurrences of $\Diamond\varphi$, where φ is a formula, by $\ulcorner$it is possible that $\varphi\urcorner$.

- Replace all occurrences of $(\exists\alpha)\varphi$, where α is a variable and φ is a formula, by $\ulcorner$some sense α is such that $\varphi\urcorner$.

- Replace all occurrences of $(\forall\alpha)\varphi$, where α is a variable and φ is a formula, by $\ulcorner$every sense α is such that $\varphi\urcorner$.

KFGS5a is a set of formulas which I define by presenting a group of inference rules which together have the property that in general a formula φ will be a member of **KFGS5a** if and only if there is a correct derivation of φ from the empty set, where a "correct" derivation is one in which each step is made in accordance with the stated rules. Let us understand by a derivation a finite sequence of consecutively numbered lines, each consisting of a formula together with the premise numbers of the line, the sequence being constructed according to the following inference rules (in these statements φ, $\varphi\prime$, ψ and $\psi\prime$ are arbitrary formulas, τ, $\tau\prime$, υ and $\upsilon\prime$ are arbitrary terms, α and β are arbitrary variables):

- Introduction of premises. Any formula may be entered on a line. As the only premise number of the new line take its line number.

- Conjunction exploitation. The formulas φ and ψ may be entered on a line if $(\varphi \wedge \psi)$ appears on an earlier line. As premise numbers of the new line take those of the earlier line.

- Conjunction introduction. The formula $(\varphi \wedge \psi)$ may be entered on a line if φ and ψ appear on earlier lines. As premise numbers of the new line take all premise numbers of those earlier lines.

- Double negation exploitation. The formula φ may be entered on a line if $\neg\neg\varphi$ appears on an earlier line. As premise numbers of the new line take those of the earlier line.

- Reductio ad absurdum. The formula $\neg\varphi$ may be entered on a line if $(\psi \wedge \neg\psi)$ appears on an earlier line. As premise numbers of the new line take those of the earlier line, with the exception (if desired) of any that is the line number of a line on which φ appears.

- Universal specification. The formula $s(\varphi, \tau, \alpha)$ may be entered on a line if $(\forall\alpha)\varphi$ appears on an earlier line. As premise numbers of the new line take those of the earlier line.

- Universal generalisation. The formula $(\forall\alpha)\varphi$ may be entered on a line if $s(\varphi, \beta, \alpha)$ appears on an earlier line and β is free neither in $(\forall\alpha)\varphi$ nor in any premise of that earlier line. As premise numbers of the new line take those of the earlier line.

- Necessitation. The formula $\Box\varphi$ may be entered on a line if φ appears on an earlier line with no premise numbers. Give the new line no premise numbers.

- Axioms. Any of the following formulas may be entered on a line.

  1. $\tau = \tau$
  2. $((\tau = \tau\prime \wedge \upsilon = \upsilon\prime) \rightarrow [\tau\upsilon] = [\tau\prime\upsilon\prime])$
  3. $((\tau = \upsilon \wedge (\tau = \tau\prime \wedge \upsilon = \upsilon\prime)) \rightarrow \tau\prime = \upsilon\prime)$
  4. $((T\tau \wedge \tau = \upsilon) \rightarrow T\upsilon)$
  5. $((\tau = \tau\prime \wedge \upsilon = \upsilon\prime) \leftrightarrow [\tau = \upsilon] = [\tau\prime = \upsilon\prime])$
  6. $(\tau = \upsilon \leftrightarrow [T\tau] = [T\upsilon])$
  7. $(([\varphi] = [\varphi\prime] \wedge [\psi] = [\psi\prime]) \leftrightarrow [(\varphi \wedge \psi)] = [(\varphi\prime \wedge \psi\prime)])$
  8. $([\varphi] = [\psi] \leftrightarrow [\neg\varphi] = [\neg\psi])$
  9. $([\lambda\alpha.[\varphi]] = [\lambda\beta.[\psi]] \leftrightarrow [(\forall\alpha)\varphi] = [(\forall\beta)\psi])$
  10. $([\varphi] = [\psi] \leftrightarrow [\Box\varphi] = [\Box\psi])$

11. $\neg[\tau = \tau'] = [T\upsilon]$
12. $\neg[\tau = \tau'] = [(\psi \wedge \psi')]$
13. $\neg[\tau = \tau'] = [\neg\psi]$
14. $\neg[\tau = \tau'] = [(\forall\beta)\psi]$
15. $\neg[\tau = \tau'] = [\Box\psi]$
16. $\neg[T\tau] = [(\psi \wedge \psi')]$
17. $\neg[T\tau] = [\neg\psi]$
18. $\neg[T\tau] = [(\forall\beta)\psi]$
19. $\neg[T\tau] = [\Box\psi]$
20. $\neg[(\varphi \wedge \varphi')] = [\neg\psi]$
21. $\neg[(\varphi \wedge \varphi')] = [(\forall\beta)\psi]$
22. $\neg[(\varphi \wedge \varphi')] = [\Box\psi]$
23. $\neg[\neg\varphi] = [(\forall\beta)\psi]$
24. $\neg[\neg\varphi] = [\Box\psi]$
25. $\neg[(\forall\alpha)\varphi] = [\Box\psi]$
26. $((\forall\alpha)\Box\varphi \rightarrow \Box(\forall\alpha)\varphi)$
27. $(\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi))$
28. $(\Box\varphi \rightarrow \varphi)$
29. $(\Diamond\varphi \rightarrow \Box\Diamond\varphi)$
30. $(T[(\varphi \wedge \psi)] \leftrightarrow (T[\varphi] \wedge T[\psi]))$
31. $(F[(\varphi \wedge \psi)] \leftrightarrow (F[\varphi] \vee F[\psi]))$
32. $(T[\neg\varphi] \leftrightarrow F[\varphi])$

33. $(F[\neg\varphi] \leftrightarrow T[\varphi])$

34. $(T[(\forall\alpha)\varphi] \leftrightarrow (\forall\alpha)T[\varphi])$

35. $(F[(\forall\alpha)\varphi] \leftrightarrow (\exists\alpha)F[\varphi])$

36. $(T[\Box\varphi] \leftrightarrow \Box T[\varphi])$

37. $(F[\Box\varphi] \leftrightarrow \Diamond F[\varphi])$

38. $(T[\tau = \upsilon] \leftrightarrow \tau = \upsilon)$

39. $(F[\tau = \upsilon] \leftrightarrow \neg\tau = \upsilon)$

40. $(T[T\tau] \leftrightarrow T\tau)$

41. $(F[T\tau] \leftrightarrow F\tau)$

42. $\neg(T\tau \wedge F\tau)$

Give the new line no premise numbers.

- Definitional equivalence. If ψ is obtained from φ by replacing an occurrence of a term or formula θ in φ by an occurrence of a term or formula to which θ is definitionally equivalent (see below), then ψ may be entered on a line if φ appears on an earlier line. As premise numbers of the new line take those of the earlier line.

For any formulas φ, ψ, terms τ, υ, and variables α, β: (a) $(\varphi \vee \psi)$ is definitionally equivalent to $\neg(\neg\varphi \wedge \neg\psi)$, and vice versa; (b) $(\varphi \rightarrow \psi)$ is definitionally equivalent to $(\neg\varphi \vee \psi)$, and vice versa; (c) $(\varphi \leftrightarrow \psi)$ is definitionally equivalent to $((\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi))$, and vice versa; (d) $(\exists\alpha)\varphi$ is definitionally equivalent to $\neg(\forall\alpha)\neg\varphi$, and vice versa; (e) $\Diamond\varphi$ is definitionally equivalent to $\neg\Box\neg\varphi$, and vice versa; (f) $[[\lambda\alpha.\upsilon]\tau]$ is definitionally equivalent to $s(\upsilon, \tau, \alpha)$, and vice versa; (g) if β is not free in τ, $[\lambda\alpha.\tau]$ is definitionally equivalent to $[\lambda\beta.s(\tau, \beta, \alpha)]$, and vice versa; (h) $F\tau$ is definitionally equivalent to $T[\neg T\tau]$, and vice versa; (i) $P\tau$ is definitionally equivalent to $(T\tau \vee F\tau)$, and vice versa; (j) if α is the first variable in the list $x_1, x_2, x_3, \ldots$ which is not free in τ, then $A_1\tau$ is definitionally equivalent to $(\forall\alpha)P[\tau\alpha]$, and vice versa; (k) for each $i \geq 1$, if α is the first variable in the list $x_1, x_2, x_3, \ldots$ which is not free in τ, then $A_{i+1}\tau$ is definitionally equivalent to $(\forall\alpha)A_i[\tau\alpha]$, and vice versa.

In order to specify formulas in a readable way, I adopt these conventions: (a) outermost parentheses may be dropped; (b) when $\wedge$ and $\vee$ are used repeatedly, the expression is grouped to the right.

1.3 Why the new tool is cool

KFGS5a recommended itself to me by its ability to shed light on ten important philosophical puzzles.

In the first place, **KFGS5a** sheds light on the problem of what the necessary and sufficient conditions for translinguistic truth are. A satisfactory solution to this problem must succesfully handle the sentence

(1.1) This is not true ,

which expresses what its first word denotes, if it expresses anything at all. Success here involves one of two things: either an argument showing that (1.1) is meaningless, or an argument showing that assigning (1.1) a meaning does not doom us to an unacceptably weak notion of translinguistic truth. **KFGS5a** allows us to deliver the latter. On the one hand, because **KFGS5a** contains the formula

(1.2)
$$[[\lambda x_1.[[\lambda x_2.[x_1[x_2 x_2]]][\lambda x_2.[x_1[x_2 x_2]]]]][\lambda x_1.[\neg T x_1]]] =$$
$$[\neg T[[\lambda x_1.[[\lambda x_2.[x_1[x_2 x_2]]][\lambda x_2.[x_1[x_2 x_2]]]]][\lambda x_1.[\neg T x_1]]]] \ ,$$

we can identify the meaning of (1.1) with the meaning of the formula

(1.3) $$\neg T[[\lambda x_1.[[\lambda x_2.[x_1[x_2x_2]]][\lambda x_2.[x_1[x_2x_2]]]]][\lambda x_1.[\neg T x_1]]]\ .$$[12]

On the other hand, even though **KFGS5a** does not contain the formula

(1.4) $$T[(1.3)] \vee F[(1.3)]\ ,$$

it contains all formulas

(1.5) $$(T[\varphi] \vee F[\varphi]) \rightarrow (T[\varphi] \leftrightarrow \varphi)\ ,$$

and so we can sacrifice the principle of bivalence without surrendering the bare minimum we need to do justice to translinguistic truth.

In the second place, **KFGS5a** sheds light on the problem of how densely populated the realm of properties is. Because in English we can build bona fide one-place predicates of indefinite complexity by means of the connectives

(1.6) $\boxed{1}$ and $\boxed{2}$

(1.7) $\boxed{1}$ or $\boxed{2}$

(1.8) it is not the case that $\boxed{1}$

(1.9) if $\boxed{1}$, then $\boxed{2}$

(1.10) $\boxed{1}$ if and only if $\boxed{2}$

(1.11) it is necessary that $\boxed{1}$

(1.12) it is possible that $\boxed{1}$,

[12]The term "$[\lambda x_1.[[\lambda x_2.[x_1[x_2x_2]]][\lambda x_2.[x_1[x_2x_2]]]]]$" denotes Curry's fixpoint combinator.

we are entitled to suppose that properties are closed with respect to the meanings of these connectives, if we think of properties as abstracta. **KFGS5a** satisfies this desideratum on two grounds. Firstly, it invites us to identify the meanings of (1.6)–(1.12) with, respectively, the meanings of the terms

(1.13) $[\lambda x_1.[\lambda x_2.[Tx_1 \wedge Tx_2]]]$

(1.14) $[\lambda x_1.[\lambda x_2.[Tx_1 \vee Tx_2]]]$

(1.15) $[\lambda x_1.[\neg Tx_1]]$

(1.16) $[\lambda x_1.[\lambda x_2.[Tx_1 \rightarrow Tx_2]]]$

(1.17) $[\lambda x_1.[\lambda x_2.[Tx_1 \leftrightarrow Tx_2]]]$

(1.18) $[\lambda x_1.[\Box Tx_1]]$

(1.19) $[\lambda x_1.[\Diamond Tx_1]]$.

Secondly, it contains the formulas

(1.20) $(\forall x_2)(\forall x_3)((A_1x_2 \wedge A_1x_3) \rightarrow A_1[\lambda x_1.[[\text{(1.13)}[x_2x_1]][x_3x_1]]]])$

(1.21) $(\forall x_2)(\forall x_3)((A_1x_2 \wedge A_1x_3) \rightarrow A_1[\lambda x_1.[[\text{(1.14)}[x_2x_1]][x_3x_1]]]])$

(1.22) $(\forall x_2)(A_1x_2 \rightarrow A_1[\lambda x_1.[\text{(1.15)}[x_2x_1]]]])$

(1.23) $(\forall x_2)(\forall x_3)((A_1x_2 \wedge A_1x_3) \rightarrow A_1[\lambda x_1.[[\text{(1.16)}[x_2x_1]][x_3x_1]]]])$

(1.24) $(\forall x_2)(\forall x_3)((A_1x_2 \wedge A_1x_3) \rightarrow A_1[\lambda x_1.[[\text{(1.17)}[x_2x_1]][x_3x_1]]]])$

(1.25) $(\forall x_2)(A_1x_2 \rightarrow A_1[\lambda x_1.[\text{(1.18)}[x_2x_1]]]])$

(1.26) $(\forall x_2)(A_1x_2 \rightarrow A_1[\lambda x_1.[\text{(1.19)}[x_2x_1]]]])$.

However, the attitude it favours is not an overindulgent one. As **KFGS5a** contains the formula

(1.27) $$\neg A_1[\lambda x_1.[\neg T[x_1 x_1]]] \,,$$

it forbids us to categorise as a property the meaning of the notorious English predicate

(1.28) $$\boxed{1} \text{ is not true of } \boxed{1} \,,$$

if we take "$\boxed{1}$ is not true of $\boxed{2}$" to be a stylistic variant of "[the outcome of the application of $\boxed{1}$ to $\boxed{2}$] is true".

In the third place, **KFGS5a** sheds light on the problem of what distinguishes denoting from expressing. As is well known, Gottlob Frege's solution to this problem is syntactic and semantic in nature: denoting and expressing are different relations because disjunctness is true not only of their first domains but also of their second domains.[13] The principle that a sentence of a natural language is never synonymous with a mere list of names or nominal expressions, which he invokes in support of this solution, is very plausible indeed. After all, it is arduous to imagine how English could evolve into a stage where "Jane, wisdom" and "Jane is wise" would say the same thing. However, one might find this solution too expensive, for it carries no guarantee that the relation between the expressandum of an English predicate and the denotatum of its nominalisation is easier to explain than the relation between a Cartesian mind and its Cartesian body. Allowing us to replace talk of two different kinds of meaning with talk of two different ways of processing one kind of meaning, **KFGS5a** makes it possible for us to use that principle to justify a purely syntactic solution. What it invites us to infer from the non-synonymity of "Jane, wisdom" and "Jane is wise" is the existence of senses x, y and z such that (a) x is both the denotatum of "wisdom" and the expressandum of "$\boxed{1}$ is wise"; (b) the outcome of the application of y

[13] See "On concept and object", in [18].

to x is the meaning of "Jane, wisdom"; (c) the outcome of the application of z to x is the meaning of "Jane is wise"; and (d) y is different from z.

In the fourth place, **KFGS5a** sheds light on the problem of how many meanings our linguistic entities have. When confronted with the English sentences

(1.29) Invisibility is invisible

(1.30) Beauty is beautiful

(1.31) Triangularity is not triangular

(1.32) The theory of relativity is not blue

(1.33) Quadratic equations cannot drink procrastination

(1.34) Pluto is invisible

(1.35) Renee Zellweger is beautiful

(1.36) The Union Jack is not triangular

(1.37) The sun is not blue

(1.38) Newborns cannot drink absinth ,

an advocate of the theory of types is left with only two alternatives: either to declare (1.29)–(1.33) meaningless, or to proclaim that together (1.29)–(1.38) are proof of the ambiguity of the English predicates

(1.39) $\boxed{1}$ is invisible

(1.40) $\boxed{1}$ is beautiful

(1.41) $\boxed{1}$ is not triangular

(1.42) $\boxed{1}$ is not blue

(1.43) $\boxed{1}$ cannot drink $\boxed{2}$.

Since **KFGS5a** is a type-free formal system, it spares us this dilemma, thereby forcing upon us neither a draconian theory of nonsense nor an arbitrary proliferation of meanings.

In the fifth place, **KFGS5a** sheds light on the substitutivity failures involving necessarily equivalent expressions in intentional contexts. Most of the possible worlds systems of intensional logic constrain us to validate the following intuitively invalid argument:

(1.44) Jane believes the proposition that the number two plus itself equals the number four

(1.45) Necessarily, the Chinese remainder theorem and the proposition that the number two plus itself equals the number four are both true or both false

(1.46) Therefore, Jane believes the Chinese remainder theorem .

KFGS5a absolves us from this bizarre commitment. Since it does not contain the formula

(1.47) $$(\forall x_1)(\forall x_2)(\Box((Tx_1 \wedge Tx_2) \vee (Fx_1 \wedge Fx_2)) \rightarrow x_1 = x_2) ,$$

we can say that “the Chinese remainder theorem” and “the proposition that the number two plus itself equals the number four” are necessarily equivalent but non-synonymous English nominal expressions, thus stopping philosophy from becoming a threat to our textbooks in number theory.

In the sixth place, **KFGS5a** sheds light on Frege’s puzzle, i.e., on the problem of how an identity sentence can be merely contingently true. Its attraction here lies in the way it helps us to improve Russell’s theory of descriptions. On the one hand, by drawing on Montague’s theory of quantifiers, we can say that the meaning of an overt or covert definite description is always a sense which results from an application of the denotatum of the term

(1.48) $$[\lambda x_1.[\lambda x_2.[(\exists x_3)((\forall x_2)(T[x_1 x_2] \leftrightarrow x_2 = x_3) \wedge T[x_2 x_3])]]] \ ,$$

thereby avoiding the undesirable syncategorematicity principle.[14] (I shall call such senses "individual concepts".) On the other hand, since making this move goes together very well with accepting

(1.49) If γ and ζ are English names or nominal expressions but neither is an overt or covert definite description, then the meaning of $\ulcorner\gamma$ is identical to $\zeta\urcorner$ is identical to [the outcome of the application of [the outcome of the application of the meaning of " $\boxed{1}$ is identical to $\boxed{2}$ " to the meaning of γ] to the meaning of ζ]

(1.50) If γ and ζ are English names or nominal expressions but only ζ is an overt or covert definite description, then the meaning of $\ulcorner\gamma$ is identical to $\zeta\urcorner$ is identical to [the outcome of the application of the meaning of ζ to [the outcome of the application of the meaning of " $\boxed{1}$ is identical to $\boxed{2}$ " to the meaning of γ]]

(1.51) If γ and ζ are English names or nominal expressions but only γ is an overt or covert definite description, then the meaning of $\ulcorner\gamma$ is identical to $\zeta\urcorner$ is identical to [the outcome of the application of the meaning of γ to [the command to output [the outcome of the application of [the outcome of the application of the meaning of " $\boxed{1}$ is identical to $\boxed{2}$ " to x_1] to the meaning of ζ], for any sense x_1 as input]]

[14] See "The proper treatment of quantification in ordinary English", in [26].

(1.52) If γ and ζ are overt or covert English definite descriptions, then the meaning of $\ulcorner\gamma$ is identical to $\zeta\urcorner$ is identical to [the outcome of the application of the meaning of γ to [the command to output [the outcome of the application of the meaning of ζ to [the outcome of the application of the meaning of " $\boxed{1}$ is identical to $\boxed{2}$ " to x_1]], for any sense x_1 as input]] ,

and **KFGS5a** contains the formula

(1.53)
$$(\forall x_1)(\forall x_2)(T[[[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]x_2] \rightarrow \Box T[[[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]x_2])$$

but not the formulas

(1.54)
$$(\forall x_1)(\forall x_2)(T[[(1.48)x_2][[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]] \rightarrow \Box T[[(1.48)x_2][[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]])$$

(1.55)
$$(\forall x_1)(\forall x_2)(T[[(1.48)x_1][\lambda x_1.[[[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]x_2]]] \rightarrow \Box T[[(1.48)x_1][\lambda x_1.[[[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]x_2]]])$$

(1.56)
$$(\forall x_1)(\forall x_2)(T[[(1.48)x_1][\lambda x_1.[[(1.48)x_2][[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]]]] \rightarrow \Box T[[(1.48)x_1][\lambda x_1.[[(1.48)x_2][[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]]]]) ,$$

there is room for maintaining that an identity sentence can be merely contingently true only if at least one overt or covert definite description occurs in it.

In the seventh place, **KFGS5a** sheds light on the problem of whether or not that-clauses are opaque. In conformity with Willard Quine's admonitions, answering in the negative carries the price of validating the intuitively invalid *de dicto* reading of the following argument:

(1.57) The Queen of England believes that England's victory in a FIFA World Cup Final took place at Wembley on July 30 1966

(1.58) England's victory in a FIFA World Cup Final is identical to the biggest scandal in the history of football

(1.59) The Queen of England believes that the biggest scandal in the history of . football took place at Wembley on July 30 1966 .[15]

With **KFGS5a** we can safely reject this view. Since "England's victory in a FIFA World Cup Final" and "the biggest scandal in the history of football" are overt or covert English definite descriptions, we can take advantage of the above discussion of Frege's puzzle to uphold

(1.60) The meaning of "England's victory in a FIFA World Cup Final took place at Wembley on July 30 1966" is identical to [the outcome of the application of the meaning of "England's victory in a FIFA World Cup Final" to the meaning of " $\boxed{1}$ took place at Wembley on July 30 1966"]

(1.61) The meaning of "England's victory in a FIFA World Cup Final is identical to the biggest scandal in the history of football" is identical to [the outcome of the application of the meaning of "England's victory in a FIFA World Cup Final" to [the command to output [the outcome of the application of the meaning of "the biggest scandal in the history of football" to [the outcome of the application of the meaning of " $\boxed{1}$ is identical to $\boxed{2}$ " to x_1]], for any sense x_1 as input]]

[15]See section 30 of [32].

(1.62) The meaning of "The biggest scandal in the history of football took place at Wembley on July 30 1966" is identical to [the outcome of the application of the meaning of "the biggest scandal in the history of football" to the meaning of " $\boxed{1}$ took place at Wembley on July 30 1966"] .

Because **KFGS5a** does not contain the formula

$$(\forall x_1)(\forall x_2)(T[[(1.48)x_1][\lambda x_1.[[(1.48)x_2][[\lambda x_1.[\lambda x_2.[x_1 = x_2]]]x_1]]]] \to (\forall x_3)[[(1.48)x_1]x_3] = [[(1.48)x_2]x_3]) \ , \tag{1.63}$$

doing so enables us to deny (1.59) and assert (1.57) and (1.58). Nevertheless, as the predicate-like status of overt or covert definite descriptions justifies us in transferring them from the first domain of the relation of denoting to the first domain of the relation of expressing, we are still free to state that there are no co-denoting English names or nominal expressions γ and ζ such that ⌜The Queen of England believes that γ took place at Wembley on July 30 1966⌝ and ⌜The Queen of England believes that ζ took place at Wembley on July 30 1966⌝ differ in truth-value.

In the eighth place, **KFGS5a** sheds light on the problem of whether or not the meanings of English predicates are functions. George Bealer thinks that an affirmative answer compels us to classify as sound the following intuitively unsound argument:

(1.64) The meaning of " $\boxed{1}$ follows Rajneesh" is identical to [the outcome of the application of the meaning of " $\boxed{2}$ follows $\boxed{1}$ " to the meaning of "Rajneesh"]

(1.65) The meaning of "Jane Fonda follows $\boxed{1}$ " is identical to [the outcome of the application of the meaning of " $\boxed{1}$ follows $\boxed{2}$ " to the meaning of "Jane Fonda"]

(1.66) The meaning of “Jane Fonda rajneeshes” is identical to [the outcome of the application of the meaning of “ $\boxed{1}$ rajneeshes” to the meaning of “Jane Fonda”]

(1.67) The meaning of “Rajneesh fondalees” is identical to [the outcome of the application of the meaning of “ $\boxed{1}$ fondalees” to the meaning of “Rajneesh”]

(1.68) [The outcome of the application of [the outcome of the application of the meaning of “ $\boxed{2}$ follows $\boxed{1}$ ” to the meaning of “Rajneesh”] to the meaning of “Jane Fonda”] is identical to [the outcome of the application of [the outcome of the application of the meaning of “ $\boxed{1}$ follows $\boxed{2}$ ” to the meaning of “Jane Fonda”] to the meaning of “Rajneesh”]

(1.69) “ $\boxed{1}$ rajneeshes” is synonymous with “ $\boxed{1}$ follows Rajneesh”

(1.70) “ $\boxed{1}$ fondalees” is synonymous with “Jane Fonda follows $\boxed{1}$ ”

(1.71) Therefore, “Jane Fonda rajneeshes” is synonymous with “Rajneesh fondalees” .[16]

KFGS5a allows us to show him wrong. Bealer’s verdict stems from his conviction that each one of these premises—with the exception of (1.69) and (1.70), which are true by his own fiat—follows immediately from the principle that predication is function application. It turns out, however, that we can preserve this principle and yet reject (1.64)–(1.67), as these four premises also depend on the controversial principle that the meaning of a proper name is its bearer. If we discard the latter in favour of the principle that the meaning of a proper

[16]See [3].

name is an individual concept, then it seems plausible for us to exchange (1.64)–(1.67) for

(1.72) The meaning of " $\boxed{1}$ follows Rajneesh" is identical to [the command to output [the outcome of the application of the meaning of "Rajneesh" to [the outcome of the application of the meaning of " $\boxed{1}$ follows $\boxed{2}$ " to x_1]], for any sense x_1 as input]

(1.73) The meaning of "Jane Fonda follows $\boxed{1}$ " is identical to [the command to output [the outcome of the application of the meaning of "Jane Fonda" to [the outcome of the application of the meaning of " $\boxed{2}$ follows $\boxed{1}$ " to x_1]], for any sense x_1 as input]

(1.74) The meaning of "Jane Fonda rajneeshes" is identical to [the outcome of the application of the meaning of "Jane Fonda" to the meaning of " $\boxed{1}$ rajneeshes"]

(1.75) The meaning of "Rajneesh fondalees" is identical to [the outcome of the application of the meaning of "Rajneesh" to the meaning of " $\boxed{1}$ fondalees"] ,

thus blocking the derivation of (1.71). In fact, because **KFGS5a** contains the formula

(1.76)
$$(\forall x_1)(\forall x_2)(\neg x_1 = x_2 \rightarrow \\ (\forall x_3)(\forall x_4)\neg[[(1.48)x_3][\lambda x_2.[[(1.48)x_4][x_1 x_2]]]] = \\ [[(1.48)x_4][\lambda x_1.[[(1.48)x_3][x_2 x_1]]]]) \ ,$$

it gives us the assurance that, together with (1.69), (1.70),

(1.77) “ $\boxed{1}$ follows $\boxed{2}$ ” is not synonymous with “ $\boxed{2}$ follows $\boxed{1}$ ”

and the assumption that the meanings of “Rajneesh” and “Jane Fonda” are individual concepts, (1.72)–(1.75) entail the negation of (1.71).

In the ninth place, **KFGS5a** sheds light on the problem of what being a tuple consists in. Obviously, an adequate solution to this problem must render true all of the English sentences

Θ_2 For all ordered pairs x_1 and x_2, x_1 is identical to x_2 if and only if the first and second ordered-pair components of x_1 are identical to, respectively, the first and second ordered-pair components of x_2

Θ_3 For all ordered triples x_1 and x_2, x_1 is identical to x_2 if and only if the first, second and third ordered-triple components of x_1 are identical to, respectively, the first, second and third ordered-triple components of x_2

Θ_4 For all ordered quadruples x_1 and x_2, x_1 is identical to x_2 if and only if the first, second, third and fourth ordered-quadruple components of x_1 are identical to, respectively, the first, second, third and fourth ordered-quadruple components of x_2

$\vdots$ $\vdots$

KFGS5a does not disappoint us here. If we identify the meanings of the English functors

Π^2_1 the first ordered-pair component of $\boxed{1}$

Π^2_2 the second ordered-pair component of $\boxed{1}$

Π^3_1	the first ordered-triple component of $\boxed{1}$
Π^3_2	the second ordered-triple component of $\boxed{1}$
Π^3_3	the third ordered-triple component of $\boxed{1}$
Π^4_1	the first ordered-quadruple component of $\boxed{1}$
Π^4_2	the second ordered-quadruple component of $\boxed{1}$
Π^4_3	the third ordered-quadruple component of $\boxed{1}$
Π^4_4	the fourth ordered-quadruple component of $\boxed{1}$
$\vdots$	$\vdots$

with, respectively, the meanings of the terms

π^2_1	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.x_2]]]]$
π^2_2	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.x_3]]]]$
π^3_1	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.[\lambda x_4.x_2]]]]]$
π^3_2	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.[\lambda x_4.x_3]]]]]$
π^3_3	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.[\lambda x_4.x_4]]]]]$
π^4_1	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.[\lambda x_4.[\lambda x_5.x_2]]]]]]$
π^4_2	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.[\lambda x_4.[\lambda x_5.x_3]]]]]]$
π^4_3	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.[\lambda x_4.[\lambda x_5.x_4]]]]]]$
π^4_4	$[\lambda x_1.[x_1[\lambda x_2.[\lambda x_3.[\lambda x_4.[\lambda x_5.x_5]]]]]]$
$\vdots$	$\vdots\,,$

we can identify the meanings of the English predicates

Δ_2	$\boxed{1}$ is an ordered pair

Δ_3	$\boxed{1}$ is an ordered triple
Δ_4	$\boxed{1}$ is an ordered quadruple
$\vdots$	$\vdots$

with, respectively, the meanings of the terms

$$
\begin{array}{ll}
\delta_2 & [\lambda x_4.[(\exists x_2)(\exists x_3) x_4 = [\lambda x_1.[[x_1 x_2] x_3]]]] \\
\delta_3 & [\lambda x_5.[(\exists x_2)(\exists x_3)(\exists x_4) x_5 = [\lambda x_1.[[[x_1 x_2] x_3] x_4]]]] \\
\delta_4 & [\lambda x_6.[(\exists x_2)(\exists x_3)(\exists x_4)(\exists x_5) x_6 = [\lambda x_1.[[[[x_1 x_2] x_3] x_4] x_5]]]] \\
\vdots & \vdots ,
\end{array}
$$

for **KFGS5a** contains all of the formulas

$$
\begin{array}{ll}
\theta_2 & (\forall x_1)(\forall x_2)((T[\rho_2 x_1] \wedge T[\rho_2 x_2]) \rightarrow (x_1 = x_2 \leftrightarrow ([\pi^2_1 x_1] = [\pi^2_1 x_2] \wedge \\
& \qquad [\pi^2_2 x_1] = [\pi^2_2 x_2]))) \\
\theta_3 & (\forall x_1)(\forall x_2)((T[\rho_3 x_1] \wedge T[\rho_3 x_2]) \rightarrow (x_1 = x_2 \leftrightarrow ([\pi^3_1 x_1] = [\pi^3_1 x_2] \wedge \\
& \qquad [\pi^3_2 x_1] = [\pi^3_2 x_2] \wedge \\
& \qquad [\pi^3_3 x_1] = [\pi^3_3 x_2]))) \\
\theta_4 & (\forall x_1)(\forall x_2)((T[\rho_4 x_1] \wedge T[\rho_4 x_2]) \rightarrow (x_1 = x_2 \leftrightarrow ([\pi^4_1 x_1] = [\pi^4_1 x_2] \wedge \\
& \qquad [\pi^4_2 x_1] = [\pi^4_2 x_2] \wedge \\
& \qquad [\pi^4_3 x_1] = [\pi^4_3 x_2] \wedge \\
& \qquad [\pi^4_4 x_1] = [\pi^4_4 x_2]))) \\
\vdots & \vdots
\end{array}
$$

It is thus possible to conceive of tuples as special kinds of senses, rather than as special kinds of sets.

Finally, **KFGS5a** sheds light on the problem of what mathematics expresses. The most standard solution to this problem is infamous for tempting us to say that, appearances notwithstanding, the English sentence

(1.78) The number zero is different from the number one

expresses in a somewhat blurred way what is perspicuously expressed by the English sentence

(1.79) The empty set is different from the singleton of the empty set .

With **KFGS5a** we can work out a solution which does not take for granted the imperialism of Zermelo-Fraenkel set theory. This solution, which represents what Michael Dummett calls "the hardheaded version of structuralism", is best illustrated through a three-stage translation. Firstly, if we agree that "a mathematical theory, even if it be number theory or analysis which we ordinarily take as intended to characterise *one* particular mathematical system, . . . always concerns all systems with a given structure",[17] we can identify the meanings of (1.78) and (1.79) with, respectively, the meanings of the English sentences

(1.78′) Every sense x_1 is such that $($if the Peano structure is true of x_1, then x_1's number zero is different from x_1's number one$)$

(1.79′) Every sense x_1 is such that $($if the Zermelo-Fraenkel structure is true of x_1, then x_1's empty set is different from x_1's singleton of x_1's empty set$)$.

[17] [15], page 296.

Secondly, if we identify the meanings of the English nominal expressions

(1.80) the Peano structure

(1.81) the Zermelo-Fraenkel structure

with, respectively, the meanings of the English nominal expressions

(1.80′) [the command to output [the thought that x_1 is an ordered triple and the first ordered-triple component of x_1 is a property and every sense x_2 is such that (if the first ordered-triple component of x_1 is true of x_2, then the first ordered-triple component of x_1 is true of the outcome of the application of the second ordered-triple component of x_1 to x_2) and the first ordered-triple component of x_1 is true of the third ordered-triple component of x_1 and x_1 satisfies the first of Peano's axioms and x_1 satisfies the second of Peano's axioms and x_1 satisfies the third of Peano's axioms], for any sense x_1 as input]

(1.81′) [the command to output [the thought that x_1 is an ordered pair and the first ordered-pair component of x_1 is a property and the second ordered-pair component of x_1 is a binary relation and x_1 satisfies the axiom of extensionality and x_1 satisfies the separation axioms and x_1 satisfies the pair set axiom and x_1 satisfies the sum set axiom and x_1 satisfies the power set axiom and x_1 satisfies the axiom of infinity and x_1 satisfies the replacement axioms], for any sense x_1 as input] ,

we can identify the meanings of (1.78′) and (1.79′) with, respectively, the meanings of the

English sentences

(1.78″) Every sense x_1 is such that $($if (1.80′) is true of x_1, then x_1's number zero is different from x_1's number one$)$

(1.79″) Every sense x_1 is such that $($if (1.81′) is true of x_1, then x_1's empty set is different from x_1's singleton of x_1's empty set$)$.

Thirdly, if we identify the meanings of the English open sentences

(1.82) x_1 satisfies the first of Peano's axioms

(1.83) x_1 satisfies the second of Peano's axioms

(1.84) x_1 satisfies the third of Peano's axioms

(1.85) x_1's number zero is different from x_1's number one

(1.86) x_1 satisfies the axiom of extensionality

(1.87) x_1 satisfies the separation axioms

(1.88) x_1 satisfies the pair set axiom

(1.89) x_1 satisfies the sum set axiom

(1.90) x_1 satisfies the power set axiom

(1.91) x_1 satisfies the axiom of infinity

(1.92) x_1 satisfies the replacement axioms

(1.93) x_1's empty set is different from x_1's singleton of x_1's empty set with, respectively, the meanings of the formulas

$$(1.82') \qquad (\forall x_2)(T[[\pi_1^3 x_1]x_2] \to \neg[[\pi_2^3 x_1]x_2] = [\pi_3^3 x_1])$$

$$(1.83') \qquad (\forall x_2)(\forall x_3)((T[[\pi_1^3 x_1]x_2] \wedge T[[\pi_1^3 x_1]x_3]) \to ([[\pi_2^3 x_1]x_2] = [[\pi_2^3 x_1]x_3] \to x_2 = x_3))$$

$$(1.84') \qquad (\forall x_2)(A_1 x_2 \to ((T[x_2[\pi_3^3 x_1]] \wedge (\forall x_3)(T[[\pi_1^3 x_1]x_3] \to (T[x_2 x_3] \to T[x_2[[\pi_2^3 x_1]x_3]]))) \to (\forall x_3)(T[[\pi_1^3 x_1]x_3] \to T[x_2 x_3])))$$

$$(1.85') \qquad \neg[\pi_3^3 x_1] = [[\pi_2^3 x_1][\pi_3^3 x_1]]$$

$$(1.86') \qquad (\forall x_2)(\forall x_3)((T[[\pi_1^2 x_1]x_2] \wedge T[[\pi_1^2 x_1]x_3]) \to ((\forall x_4)(T[[\pi_1^2 x_1]x_4] \to (T[[[\pi_2^2 x_1]x_4]x_2] \leftrightarrow T[[[\pi_2^2 x_1]x_4]x_3])) \to x_2 = x_3))$$

(1.87′) $$(\forall x_2)(\forall x_3)((A_1 x_2 \wedge T[[\pi_1^2 x_1]x_3]) \rightarrow (\exists x_4)(T[[\pi_1^2 x_1]x_4] \wedge (\forall x_5)(T[[\pi_1^2 x_1]x_5] \rightarrow (T[[[\pi_2^2 x_1]x_5]x_4] \leftrightarrow (T[[[\pi_2^2 x_1]x_5]x_3] \wedge T[x_2 x_5]))))))$$

(1.88′) $$(\forall x_2)(\forall x_3)((T[[\pi_1^2 x_1]x_2] \wedge T[[\pi_1^2 x_1]x_3]) \rightarrow (\exists x_4)(T[[\pi_1^2 x_1]x_4] \wedge (\forall x_5)(T[[\pi_1^2 x_1]x_5] \rightarrow (T[[[\pi_2^2 x_1]x_5]x_4] \leftrightarrow (x_5 = x_2 \vee x_5 = x_3)))))$$

(1.89′) $$(\forall x_2)(T[[\pi_1^2 x_1]x_2] \rightarrow (\exists x_3)(T[[\pi_1^2 x_1]x_3] \wedge (\forall x_4)(T[[\pi_1^2 x_1]x_4] \rightarrow (T[[[\pi_2^2 x_1]x_4]x_3] \leftrightarrow (\exists x_5)(T[[\pi_1^2 x_1]x_5] \wedge T[[[\pi_2^2 x_1]x_5]x_2] \wedge T[[[\pi_2^2 x_1]x_4]x_5]))))))$$

(1.90′)
$$
\begin{aligned}
&(\forall x_2)(T[[\pi^2_1 x_1]x_2] \to \\
&\quad (\exists x_3)(T[[\pi^2_1 x_1]x_3]\wedge \\
&\quad\quad (\forall x_4)(T[[\pi^2_1 x_1]x_4] \to \\
&\quad\quad\quad (T[[[\pi^2_2 x_1]x_4]x_3] \leftrightarrow \\
&\quad\quad\quad (\forall x_5)(T[[\pi^2_1 x_1]x_5] \to \\
&\quad\quad\quad\quad (T[[[\pi^2_2 x_1]x_5]x_4] \to \\
&\quad\quad\quad\quad T[[[\pi^2_2 x_1]x_5]x_2]))))))
\end{aligned}
$$

(1.91′)
$$
\begin{aligned}
&(\exists x_2)(T[[\pi^2_1 x_1]x_2]\wedge \\
&\quad (\exists x_3)(T[[\pi^2_1 x_1]x_3]\wedge \\
&\quad\quad (\forall x_4)(T[[\pi^2_1 x_1]x_4] \to \\
&\quad\quad\quad \neg T[[[\pi^2_2 x_1]x_4]x_3])\wedge \\
&\quad\quad T[[[\pi^2_2 x_1]x_3]x_2])\wedge \\
&\quad (\forall x_3)(T[[\pi^2_1 x_1]x_3] \to \\
&\quad\quad (T[[[\pi^2_2 x_1]x_3]x_2] \to \\
&\quad\quad (\exists x_4)(T[[\pi^2_1 x_1]x_4]\wedge \\
&\quad\quad\quad (\forall x_5)(T[[\pi^2_1 x_1]x_5] \to \\
&\quad\quad\quad\quad (T[[[\pi^2_2 x_1]x_5]x_4] \leftrightarrow \\
&\quad\quad\quad\quad (T[[[\pi^2_2 x_1]x_5]x_3]\vee \\
&\quad\quad\quad\quad x_5 = x_3)))\wedge \\
&\quad\quad\quad T[[[\pi^2_2 x_1]x_4]x_2]))))
\end{aligned}
$$

(1.92′)
$$
\begin{aligned}
&(\forall x_2)(A_2x_2 \rightarrow \\
&\quad((\forall x_3)(T[[\pi^2_1x_1]x_3] \rightarrow \\
&\qquad(\exists x_4)(T[[\pi^2_1x_1]x_4] \wedge \\
&\qquad\quad T[[x_2x_3]x_4] \wedge \\
&\qquad\quad(\forall x_5)(T[[\pi^2_1x_1]x_5] \rightarrow \\
&\qquad\qquad(T[[x_2x_3]x_5] \rightarrow \\
&\qquad\qquad x_5 = x_4)))) \rightarrow \\
&\quad(\forall x_3)(T[[\pi^2_1x_1]x_3] \rightarrow \\
&\qquad(\exists x_4)(T[[\pi^2_1x_1]x_4] \wedge \\
&\qquad\quad(\forall x_5)(T[[\pi^2_1x_1]x_5] \rightarrow \\
&\qquad\qquad(T[[[\pi^2_2x_1]x_5]x_4] \leftrightarrow \\
&\qquad\qquad(\exists x_6)(T[[\pi^2_1x_1]x_6] \wedge \\
&\qquad\qquad\quad T[[[\pi^2_2x_1]x_6]x_3] \wedge \\
&\qquad\qquad\quad T[[x_2x_6]x_5])))))))
\end{aligned}
$$

(1.93′)
$$
\begin{aligned}
&(\exists x_2)(T[[\pi^2_1x_1]x_2] \wedge \\
&\quad(\forall x_3)(T[[\pi^2_1x_1]x_3] \rightarrow \\
&\qquad\neg T[[[\pi^2_2x_1]x_3]x_2]) \wedge \\
&\quad(\exists x_3)(T[[\pi^2_1x_1]x_3] \wedge \\
&\qquad(\forall x_4)(T[[\pi^2_1x_1]x_4] \rightarrow \\
&\qquad\quad(T[[[\pi^2_2x_1]x_4]x_3] \leftrightarrow \\
&\qquad\quad x_4 = x_2)) \wedge \\
&\qquad\neg x_2 = x_3)) \,,
\end{aligned}
$$

we can identify the meanings of (1.78′′) and (1.79′′) with, respectively, the meanings of the formulas

$$(1.78''') \qquad (\forall x_1)(T[[\lambda x_1.[T[\delta_3 x_1] \wedge$$
$$A_1[\pi^3_1 x_1] \wedge$$
$$(\forall x_2)(T[[\pi^3_1 x_1]x_2] \rightarrow T[[\pi^3_1 x_1][[\pi^3_2 x_1]x_2]]) \wedge$$
$$T[[\pi^3_1 x_1][\pi^3_3 x_1]] \wedge$$
$$(1.82') \wedge$$
$$(1.83') \wedge$$
$$(1.84')]]x_1] \rightarrow$$
$$(1.85'))$$

$$(1.79''') \qquad (\forall x_1)(T[[\lambda x_1.[T[\delta_2 x_1] \wedge$$
$$A_1[\pi^2_1 x_1] \wedge$$
$$A_2[\pi^2_2 x_1] \wedge$$
$$(1.86') \wedge$$
$$(1.87') \wedge$$
$$(1.88') \wedge$$
$$(1.89') \wedge$$
$$(1.90') \wedge$$
$$(1.91') \wedge$$
$$(1.92')]]x_1] \rightarrow$$
$$(1.93'))\ .$$

Besides refraining from privileging Zermelo-Fraenkel set theory over Peano arithmetic, this translation guarantees the non-synonymity of (1.78) and (1.79), for **KFGS5a** contains the formula

(1.94) $$\neg[(1.78''')] = [(1.79''')]\ .$$

As to the main complaint against hardheaded structuralism—“that, while it may not be for mathematics to say whether or not there exist any systems exemplifying the structures that it studies, the subject would appear futile unless there was a strong chance that they would exist”[18]—those who are in search of a clue as to how to counter it without resorting to an overinflated background ontology might profit from exploring the fact that **KFGS5a** contains all formulas

(1.95) $$\neg[(\forall\alpha)(\varphi \to \psi)] = [(\forall\alpha)(\varphi \to \neg\psi)]\ ,$$

which suggests that the possible or actual existence of Peano systems is not a necessary condition for (1.78) to be more fruitful than the English sentence

(1.96) The number zero is not different from the number one .

This I shall not do in these pages, though.

[18] [15], page 296.

Chapter 2

Chapter 3

Chapter 4

Appendix

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