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TECHNICAL REPORT NO. 46

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REVISED NOVEMBER 1976

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Abstract

This paper gives a brief introduction to the logic of Relevant Implication. It argues that the notion of implication needed for question-answering systems is more accurately modelled by Relevant Implication than by traditional material implication. It presents two examples in which material implication leads to pathologies avoidable by the use of relevant implication. One example is the introduction of universes of discourse, and the second involves the deduction of new rules of inference.

1. Why Formal Systems?

Before beginning the discussion, let us give an informal description of what we mean by a formal system. A formal system consists of a set of formal objects, e.g., formulas (usually represented as finite strings), a distinguished subset called axioms, and a set of rules of inference. A formal object A is a theorem of the system (we write $\vdash A$) if it is derivable from the axioms by successive applications of the rules of inference. This should be familiar material, which we restate only to indicate the wide scope of the notion. Useful formal systems have been constructed in which the formal objects are not the "usual" logical formulas but strings (Hopcroft, J., and Ullman, J. [10]), pairs of formulas (Kleene [13]), programs (Milner [14]), or even proofs themselves (Fitch [7]).

This flexibility is the reason that formal systems should be of interest to the non-mathematician and to the AI practitioner in particular. In each case, the formal objects are the objects under study in some application, and a formal object is a theorem if and only if it has some property which is important for that application, e.g.,

- (i) W is a derivable string in a grammar G
- (ii) A logically implies B
- (iii) The function defined by program S is a subfunction of that defined by T
- (iv) P is a valid proof

If one has an informal notion, one may formalize it (pun intended) by writing down some of its local properties as axioms or rules of inference and then relying on deduction as a global structure. It may be that the finished system no longer has some desired properties of the original informal notion. In that case, one must patch the axioms or rules. To find the offending axiom requires considerable intellectual effort, just as it does to debug a program.

The classical propositional calculus, for example, is intended to model among other things the informal notion of an argument. In the early portion of the twentieth century, philosophers discovered arguments which they believed to be nonvalid, but which the classical propositional calculus deemed valid. After analyzing these "bugs," they modified the formal system and arrived at a formal system which more closely matched their conception of an argument: the intuitionistic propositional calculus. Similar criticisms resulted in the creation of the various modal logics (Hughes and Cresswell [11]).

This analysis does not necessarily assert that classical logic is "wrong"; we may take it to mean that the informal notion we are interested in does not coincide with the notion defined by the original formal system. Logicians have studied (in more or less detail) a wide range of formal systems, and, if there is an informal notion we wish to formalize, there may already be a known formal system embodying our concerns.

In this paper, we will argue that classical material implication is not an exact model of our informal notion of "implication." We will present a brief account of the work of Anderson, Belnap, and their colleagues [2] on the logic of Relevant Implication.

Last, we will argue that the formal notion of Relevant Implication is better than material implication as a model of what "implication" ought to mean in a question-answering system.

This paper is not intended to be a brief for the exclusive use of logic as the top level structure for a question-answering system. Instead, we mean to point out that even a "procedural" top-level in fact constitutes a formal system (albeit an extremely complicated one). The study of simpler formal systems should therefore give some guidelines for the desired behavior of the kind of system (either "procedural" or "formal") which we eventually implement on computing machinery.

2. Implication vs. Material Implication

The suspicion that material implication is a less than perfect model of the intuitive notion of implication is not new, for it is embodied in the names of certain classically valid wffs, which are known collectively as the Paradoxes of Implication (Hughes and Cresswell [11]; Ambrode and Lazerowitz [1]). Here is a list of such "strange" wffs, with paraphrases:

(1) $A \supset (B \supset A)$ "If a thing is true, then anything implies it" (this is called Positive Paradox).

(2) $A \supset (B \vee \sim B)$ "Anything implies an analytic truth" (a corollary of Positive Paradox).

(3) $(A \& \sim A) \supset B$ "A false statement implies any statement."

Even those who do not find the preceding wffs unintuitive often balk at

(4) $(A \supset B \& C) \vee (B \supset A \& C) \vee (C \supset A \& B)$ "Given any three statements, there is one which implies the other two."

This last formula says in effect that there are only two propositions in the world: the true one and the false one. If one accepts that, then A ⊃ B is the true proposition if and only if A is the false proposition or B is the true one.

Luckily, (4) and its generalizations are false in the intuitionistic calculus and its cousins, all of which insist on the existence of infinitely many propositions (and in which implication is not expressible in terms of the other propositional connectives). Less luckily, (1), (2) and (3) are still valid intuitionistically, so we need some additional analysis of why we object to (1)-(3).

One way to attack (1)-(3) is to note that the linguistic usage of "if...then" or "therefore" (which "⊃" is designed to model) usually involves an element of causality. As Anderson and Belnap say:

> Of course we *can* say "Assume that snow is puce. Seven is a prime number." But if we say "Assume snow is puce. *It follows that* (or *consequently*, or *therefore*, or *it may be validly assumed that*) seven is a prime number," we have simply spoken falsely (Anderson & Belnap [2], p. 14, italics in original).

> Imagine, if you can, a situation as follows. A mathematician writes a paper on Banach spaces, and after proving a couple of theorems he concludes with a conjecture. As a footnote to the conjecture, he writes: "In addition to its intrinsic interest this conjecture has connections with other parts of mathematics which might not immediately occur to the reader. For example, if the conjecture is true, then the first order functional calculus is complete; whereas if it is false, then it implies that Fermat's last conjecture is correct." The editor replies that the paper is obviously acceptable, but he finds the final footnote perplexing; he can see no connection whatever between the conjecture and the "other parts" of mathematics, and none is indicated in the footnote. So the mathematician replies, "Well, I was using 'if...then--' and 'implies' in the way that logicians have claimed I was: the first order functional calculus *is* complete, and necessarily so, so

> anything implies that fact -- and if the conjecture is false it is presumably impossible, and hence implies anything. And if you object to this usage, it is simply because you have not understood the technical sense of 'if...then--' worked out so nicely for us by logicians." ([2], p. 17)

They conclude:

> Material implication is <u>not</u> a "kind" of implication, or so we hold; it is no <u>more</u> a kind of implication than a blunderbuss is a kind of bus. ([2], p. 5)

Without taking quite so radical a philosophical stance, we may still adopt much of their analysis of what went wrong in (1)-(3) and the other examples. The problem is, baldly, that the primeness of seven has nothing to do with puceness (pucity?) of snow. If "A implies B" is to involve an element of causality, then surely A and B must have something in common. In (2) and (3), the antecedent clearly has nothing to do with the consequent. In (1), we are asked to assert, upon the truth of A, another implication in which the two sides have no element in common.

Of course, merely insisting that the two sides have some element in common is not sufficient: $\sim(A \supset A) \supset (A \supset A)$ is a theorem of the classical system, but this is also implausible as an implication. Since we already know that the conclusion $A \supset A$ is true -- why bother to assert the theorem? This leads to a second attack on (1)-(3). In a question-answering system, an implication has imperative as well as declarative content: an implication ought to be a useful inference rule (Hewitt [9, Ch. 1]). (Some classically-based theorem provers already discriminate between the imperative content of $\sim A \vee B$ and that of $A \supset B$ [15].)

A "fail-safe" heuristic, avoiding the dual quagmires of causality and plausible inference, but still rejecting all of the pathological examples cited, is: "do not assert 'A implies B' unless the hypothesis A was actually used in the proof of B." A formal system embodying this heuristic is the system R of Relevant Implication of Andrson and Belnap [2].

3. Relevance Logic

We will compare the logic of relevant implication with standard logic using the natural deduction notation of Fitch [7] (a similar system is that of Kalish and Montague [12]). Following Anderson and Belnap [2], we style the standard Fitch system $F\underset{\sim}{H}$ and the relevant Fitch system $F\underset{\sim}{R}$. In order to avoid details which are irrelevant to this paper, we will discuss only the fragments $F\underset{\sim}{H}_{\supset\wedge}$ and $F\underset{\sim}{R}_{\to\wedge}$ in which the indicated connectives are the only connectives. The full $F\underset{\sim}{R}$ system is presented in Anderson and Belnap [2], pp. 346-348.

A proof in Fitch's notation is a set of properly nested subproofs, the outermost of which is called categorical, the others being called hypothetical. Each subproof consists of an ordered set of wffs and subproofs introduced according to the rules of the system. In the system $F\underset{\sim}{R}_{\to\wedge}$, each wff has a set associated with it. We will use lowercase Greek letters to represent these sets. If A is a wff with associated set α, we will write the pair as

$$A,\alpha$$

The universe whose elements comprise these sets is arbitrary but presumed "large enough" for any proof. It is standard to use the natural numbers. The last element of a categorical subproof will be a wff which is thereby proved to be a theorem of the system. In $FR_{\sim\to\wedge}$ the set associated with this wff will be the empty set. The Fitch-style system consists of a set of rules showing how proofs may be extended. These are shown below for $FH_{\sim\supset\wedge}$ and for $FR_{\sim\to\wedge}$ for comparison.

(1) To introduce a new hypothetical subproof

```
in FH~⊃∧  | A  hyp          in FR~→∧  | A,{k}    hyp
          |---                        |------
          |                           |
```

where k is a singleton set whose element, k, has never before appeared in the proof.

(2) To repeat a wff within a subproof

```
in FH~⊃∧  | A               in FR~→∧  | A,α
          | .                         | .
          | .                         | .
          | .                         | .
          | A rep                     | A,α   rep
          |                           |
```

(3) To reiterate a wff from a subproof into a nested subproof

```
in FH~⊃∧  | A               in FR~→∧  | A,α
          | .                         | .
          | .                         | .
          | .                         | .
          |  | B    hyp               |  | B,{k}  hyp
          |  |---                     |  |------
          |  | .                      |  | .
          |  | .                      |  | .
          |  | .                      |  | .
          |  | A    reit              |  | A,α    reit
          |  |                        |  |
```

(4) To derive a wff from two others in the same subproof, eliminating an implication (entailment) connective. (Compare Modus Ponens)

in $FH_{\sim\supset\wedge}$

$$\begin{array}{|ll} \vdots & \\ A & \\ \vdots & \\ A\supset B & \\ B & \supset E \end{array}$$

in $FR_{\sim\rightarrow\wedge}$

$$\begin{array}{|ll} \vdots & \\ A,\alpha & \\ \vdots & \\ A\rightarrow B,\beta & \\ B,\alpha\cup\beta & \rightarrow E \end{array}$$

(5) To terminate a subproof and introduce an implication (entailment) formula into the immediately outer subproof. (Compare the Deduction Theorem)

in $FH_{\sim\supset\rightarrow}$

$$\begin{array}{|ll} \begin{array}{|ll} \underline{A} & \text{hyp} \\ \vdots & \\ B & \end{array} & \\ A\supset B & \supset I \end{array}$$

in $FR_{\sim\rightarrow\wedge}$

$$\begin{array}{|ll} \begin{array}{|ll} \underline{A,\{k\}} & \text{hyp} \\ \vdots & \\ B,\beta & \end{array} & \\ A\rightarrow B,\beta-\{k\} & \rightarrow I \end{array}$$

where $\rightarrow I$ is only allowed if $k \in \beta$.

(6) To derive a wff from another in the same subproof, eliminating a conjunction.

in $FH_{\sim\supset\wedge}$

$$\begin{array}{|ll} \vdots & \\ A\wedge B & \\ A & \wedge E \end{array}$$

and

$$\begin{array}{|ll} \vdots & \\ A\wedge B & \\ B & \wedge E \end{array}$$

in $FR_{\sim\rightarrow\wedge}$

$$\begin{array}{|ll} \vdots & \\ A\wedge B,\alpha & \\ A,\alpha & \wedge E \end{array}$$

and

$$\begin{array}{|ll} \vdots & \\ A\wedge B,\alpha & \\ B,\alpha & \wedge E \end{array}$$

(7) To derive a wff from two others in the same subproof, introducing a conjunction.

in $FH_{\sim\supset\wedge}$

$$\begin{array}{|ll} \vdots & \\ A & \\ \vdots & \\ B & \\ A\wedge B & \wedge I \end{array}$$

in $FR_{\sim\rightarrow\wedge}$

$$\begin{array}{|ll} \vdots & \\ A,\alpha & \\ \vdots & \\ B,\alpha & \\ A\wedge B,\alpha & \wedge I \end{array}$$

As an example, we will show the proofs of the law of transitivity in both systems:

in $\mathrm{FH}_{\supset\wedge}$

```
|A⊃B                    hyp
| |B⊃C                  hyp
| |A⊃B                  reit
| | |A                  hyp
| | |A⊃B                reit
| | |B                  ⊃E
| | |B⊃C                reit
| | |C                  ⊃E
| |A⊃C                  ⊃I
|(B⊃C)⊃(A⊃C)            ⊃I
(A⊃B)⊃[(B⊃C)⊃(A⊃C)]     ⊃I
```

in $\mathrm{FR}_{\to\wedge}$

```
|A→B,{1}                      hyp
| |B→C,{2}                    hyp
| |A→B,{1}                    reit
| | |A,{3}                    hyp
| | |A→B,{1}                  reit
| | |B,{1,3}                  →E
| | |B→C,{2}                  reit
| | |C,{1,2,3}                →E
| |A→C,{1,2}                  →I
|(B→C)→(A→C),{1}              →I
(A→B)→[(B→C)→(A→C)],{}        →I
```

Note that the effect of the sets used in $\mathrm{FR}_{\to\wedge}$ is to record for each wff in the proof, those hypotheses that were really used in the derivation of the wff. The restriction on the use of →I ensures that whenever A→B,α appears in a proof, A is <u>relevant</u> to the deduction of B under the hypotheses noted by α. To illustrate the effect of this, we show the proofs of the paradoxes of implication in $\mathrm{FH}_{\supset\wedge}$, and next to them the parallel derivations in $\mathrm{FR}_{\to\wedge}$ as far as they can legally be carried out.

in $\mathrm{FH}_{\supset\wedge}$

```
|A              hyp
| |B            hyp
| |A            reit
|B⊃A            ⊃I
A⊃(B⊃A)         ⊃I
```

in $\mathrm{FR}_{\supset\to\wedge}$

```
|A,{1}          hyp
| |B,{2}        hyp
| |A,{1}        reit
|
```

```
|A              hyp
| |B            hyp
| |B            rep
|B⊃B            ⊃I
A⊃(B⊃B)         ⊃I
```

```
|A,{1}          hyp
| |B,{2}        hyp
| |B,{2}        rep
|B→B,{}         →I
```

Note that in both examples, the restriction on $\to$I prevents continuing the parallel derivation and underscores the feeling that something irrelevant is happening in the proof in $FH_{\sim\supset\wedge}$. While these examples do not constitute a proof, it can be shown that neither $A\to(B\to A)$ nor $A\to(B\to B)$ is provable in $FR_{\sim\to\wedge}$.

The restriction on $\wedge$I may seem surprisingly strict. One might think that the following rule would be correct

```
| .
| .
| .
| A,α
| .
| .
| .
| B,β
| A∧B,α∪β      ∧I
```

However, this would allow the following proof:

```
| A,{1}                  hyp
|----------
|    | B,{2}             hyp
|    |--------
|    | A,{1}             reit
|    | B,{2}             rep
|    | A∧B,{1,2}         ∧I
|    | A,{1,2}           ∧E
|  B→A,{1}               →I
A→(B→A),{}               →I
```

The restriction on $\wedge$I prevents such gratuitous exchanges of the index sets.

4. Some Connections to Question-Answering Systems

4.1 Universes of Discourse

Shapiro [16] discusses the use of categorization, source, and universe of discourse indications with information stored in a

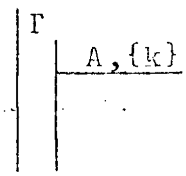
Question-Answering System (QAS)†. He points out that two subsets of the information in the data base

> ...may contain contradictory information either because two users have conflicting beliefs or because two fields [of inquiry] have different logical systems. These apparent conflicts will not matter if the simultaneous use of the two subsets is avoided by using categorization pointers...Another use for the source pointer is to keep track of the assumption and deduction rules upon which a deduced substructure is based. The reason for doing this is that there is nothing to stop a user from entering inconsistent information into the data base. He may do this unknowingly, and may discover contradictions at a later time. It would then be useful to discover the source of the contradiction and remove it. (pp. 107-109)

We now see that source pointers are equivalent to the elements of the singleton sets introduced when new subproofs are begun, and that categorization and universe of discourse indicators are equivalent to the union of the sets associated with the hypothesis of a subproof and the hypotheses of all higher subproofs. The universes of discourse form a hierarchy of contexts. The rules of $FR_{\sim\rightarrow\wedge}$ show us how to limit our deductions and in which contexts a deduced assertion may be considered to lie if we are to limit our QAS to "relevant" assertions.

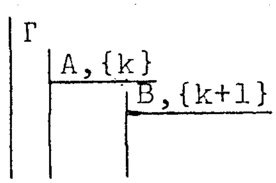
Notice that the rules $\wedge$I and $\wedge$E show us that there are two different ways of introducing a new assertion into a QAS and that the choice determines what future assertions may be deduced. Here is a typical context:

†A similar proposal for partitioning the data base into sub-data bases is in Hendrix [8].

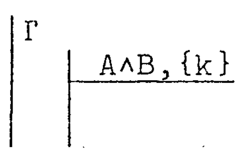


Here A, with source pointer {k}, represents the hypothesis of the innermost context layer, and Γ represents the set of formulas in the outer layers, all of which may be reiterated into the current context. The "universe of discourse" is $\{k\} \cup \{i \mid i \in \alpha$ and α is the subscript of some formula in $\Gamma\}$.

Now consider adding a new piece of information B. We may do this in two ways: we can add a new context layer B,{k+1} as follows:



or we may say, "I didn't tell you the complete truth before. Rather than 'A,{k}', the assumption should have been 'A∧B,{k}.' " The second choice yields



The rules of reiteration and ∧E ensure that in both cases A and B (as well as the formulas in Γ) may each be asserted in the current context. The two arrangements differ, however, in what implications may be deduced for use in the context Γ. Let us imagine, for example, that we may deduce a formula C using only B and some formulas from Γ. The proofs would look as follows, where $k \notin \gamma$:

```
| Γ                                   | Γ
|    | A, {k}                         |  | A∧B, {k}
|    |----------                      |  |-----------
|    |     | B, {k+1}                 |  | B, {k}            ∧E
|    |     |----------                |  | .
|    |     | .                        |  | .
|    |     | .                        |  | .
|    |     | .                        |  | C, γ∪{k}
|    |     | C, γ∪{k+1}               | A∧B→C, γ
|    | B→C, γ                         |
```

In the first case we deduced B→C in A's context, but we may not deduce A→(B→C) or B→(A→C) in the context Γ, because neither "A→..." is a relevant implication (although it is easy to show that B→C, may be derived in Γ). In the second case, however, we may deduce A∧B→C in the context Γ. This may be interpreted as saying A and B jointly imply C relevantly; A∧B is regarded as a "packet" which is left as an unanalyzed whole for determining relevance. This is the way information is usually treated; we normally write things like "G is a group → P" without regard for the relevance of all the pieces of information that make up the packet "G is a group."

We hope that this discussion will shed some light on the question of when to introduce a new context layer. The set associated with each proposition serves simultaneously the purpose of the source pointer suggested by Shapiro [16] and the purpose of $F\underset{\sim}{R}_{\to\wedge}$ to avoid the paradoxes of implication. Each derived piece of information will carry with it an indication of the assumptions under which it was derived, and all data with the same set of indices are thereby in the same universe of discourse. The rules of $F\underset{\sim}{R}_{\to\wedge}$ will prevent us from using the deduction rules of one universe incorrectly on the data of another.

An alternate view of this situation is that a QAS contains a data base, DB, of assertions of the form A,ϕ,α, where A is some formula, $\phi\varepsilon\{0,1\}$, and α is a set. The rules for adding assertions to DB are:

1. (hyp) $A,0,\{k\}$ may be added to DB as long as $\{k\}$ is a singleton set such that no assertion of the form $B,0,\{k\}$ is already in DB.

2. (add) $A,0,\{k\}$ may be removed from DB and replaced by $A\wedge B,0,\{k\}$.

3. ($\rightarrow$E) If A,ϕ,α ε DB and $A\rightarrow B,\phi,\beta$ ε DB, then $B,1,\alpha\cup\beta$ may be added to DB.

4. ($\rightarrow$I) If $A,0,\{k\}$ ε DB and B,ϕ,β ε DB and $k\varepsilon\beta$, then $A\rightarrow B,1,\beta-\{k\}$ may be added to DB.

5. ($\wedge$E) If $A\wedge B,\phi,\alpha$ ε DB, then $A,1,\alpha$ may be added to DB and $B,1,\alpha$ may be added to DB.

6. ($\wedge$I) If A,ϕ,α ε DB and B,ϕ,α ε DB, then $A\wedge B,1,\alpha$ may be added to DB.

In this scheme, all assertions of the form $A,0,\alpha$ are hypotheses entered by the user and all assertions of the form $A,1,\alpha$ are assertions which have been derived under the set of assumptions $\{(B,0,\{k\})|k\varepsilon\alpha\}$. A context is a set γ and is said to contain the set of assertions $\{(A,\phi,\alpha)|\alpha\subseteq\gamma\}$. For any contexts δ,γ such that $\gamma\subseteq\delta$, δ is a sub-context of γ and γ is an enclosing context of δ.

It is worthwhile considering the status of assertions $C,1,\alpha$ that were derived from the hypothesis $A,0,\{k\}$ before the latter was updated by use of the add rule to A B,0,{k}. If $k\varepsilon\alpha$, the formula C must have been derived using the formula A. This derivation is still valid since $A,1,\{k\}$ is derivable by the rule $\wedge$E. If $k\notin\alpha$, C might be of the form A→D. In this case, C is still valid since A,0,{j} could be introduced by hyp, D,1,α∪{j} could be derived as D,1,α∪{k} was earlier, and A→D,1,α derived by →I. All contexts containing j can now be discarded, having served their purpose. The only other case is that C derives from a formula of the form A→D, in which case, since A→D is still valid, so is C. So we see that all assertions derived before an application of add remain valid afterward.

4.2 The Garden of Eden Path Phenomenon

There are two ways in which stored data, including stored deduction rules, may be combined to derive new deduction rules in $FR_{\sim\to\wedge}$. Using →E, an assertion of the form P,ϕ,α may be combined with an assertion of the form P→(Q→R),ϕ,β to produce a deduction rule of the form Q→R,1,α∪β. This latter deduction rule holds under the assumptions α and β. That is, it holds in those worlds in which both the assumptions α and the assumptions β hold.

The other way to derive new deduction rules is based on →I. The following discussion of this method will demonstrate a real benefit in the use of relevance logic to design Question-Answering Systems. Consider a universe of discourse, α, and the new, hypothetical world produced by assuming P,0,{p}. If, in this hypothetical world, we can derive Q,1,α∪{p}, we can then derive the new deduction rule P→Q,1,α in the original universe by use of →I. This is a productive rule in the sense that if we later learn that P,ϕ,β is true, we can derive Q,1,α∪β. How might this use of →I actually be programmed in a QAS? We might take a stored assertion and produce the hypothesis, P, by generalization or by replacing certain constants by others. For example, a cognitive mobile robot might take the fact that children can move from one location to another to produce the hypothetical situation of a child moving in front of the robot. Use of $\rightarrow I$ will, hopefully, produce a plan which could be used promptly if the situation ever did result. Unfortunately, if the rules of $FH_{\supset\wedge}$ were used, such meaningless plans as "if a child moves in front of me than I can push a block into another room by first positioning myself behind the block" might be derived. The rules of $FR_{\rightarrow\wedge}$ are precisely the right ones to ensure that any derived plans are in fact relevant to the hypothetical situation.

Another problem that must be faced in deriving plans in hypothetical worlds is how much time or other resources should be spent in each such endeavor. A fixed bound seems to be insufficiently flexible. A more acceptible solution (assuming the problem of recognizing "interesting" facts has been at least partially solved) would be, "Keep going if you are producing interesting facts and stop if you are producing uninteresting ones." Certainly this runs the risk of being "led down the garden path" by a hypothetical situation that will never come about. The worst garden path would be the "Garden of Eden Path" produced by classical rules from a hypothetical that implies a contradiction, since all sorts of wonderful facts would then be derivable. In classical logic the only way to avoid Garden of Eden paths is to check for the consistency of each hypothetical situation -- a prohibitively costly method at best.

The rules of relevance logic would at least prevent a standard garden path from becoming a Garden of Eden path with the resultant waste of computer resources.

4.3 The Other Connectives

Our discussion has involved only the connectives $\rightarrow$ and $\wedge$, because we were concentrating on the issues of the deduction and use of deduction rules and on the conjoining of new assertions into a data base. A discussion of how $\underset{\sim}{FR}_{\rightarrow\wedge}$ is extended to the full $\underset{\sim}{FR}$ is in Anderson and Belnap [2], pp. 346-348. A related system, called the logic of first degree entailment is developed in Anderson and Belnap [2] Chap. III. This discusses formulas of the form $A\rightarrow B$, where A and B are formulas containing any truth functional connectives, but no arrows. Its application to Question-Answering Systems is discussed in Belnap [4,5], Bechtel [3] and Bechtel and Shapiro [4].

5. Conclusions

Following a general discussion of the significance of non-standard logics to AI practitioners, we argued that material implication has serious deficiencies as a model of the kind of implication needed for question-answering systems. We gave a brief discussion of a system $\underset{\sim}{FR}_{\rightarrow\wedge}$ of relevant implication. We presented two examples where the pathologies generated by material implication were rectified by the use of relevant implication. It is important to remember that a logical analysis of question-answering and language understanding systems is not relevant only when they are being driven by formal theorem provers. The design of any inference system entails a commitment to some system of logic. We believe that relevant implication or some variant thereof will be a useful tool in question-answering systems, natural language understanding systems and similar applications.

6. Acknowledgements

The authors thank J. Michael Dunn for his explanations of these logics. N. Belnap provided additional encouragement.

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