Closed books and laptops, one notes sheet allowed, closed neighbors, 75 minutes. Do all five problems (note problem 4 is very short) on these exam sheets. Please show all your work—this may help for partial credit. The exam totals 100 pts., subdivided as shown.

**Notation:** All problems on this exam use alphabet $\Sigma = \{a, b\}$. $\#c(x)$ stands for the number of occurrences of the character $c$ in the string $x$.

(1) (21 pts.)

Let $A = \{ x \in \{a, b\}^* : bb$ is a substring of $x$ and $\#a(x)$ is even $\}$. Design a deterministic finite automaton (DFA) $M$ such that $L(M) = A$. A node-arc diagram that shows the start and final states clearly is good enough—you need not write out tables or “$M = (Q, \Sigma, \delta, s, F)\ldots$” etc.

(Important time-saver: Design $M$ by theory and strategy—trial-and-error may take too long. The correctness of your $M$ must be clear either from the theory you applied or from comments on how $M$ works.)
Let $N$ be the NFA defined by $N = (Q, \Sigma, \delta, s, F)$ with $Q = \{1, 2, 3\}$, $\Sigma = \{a, b\}$, $s = 1$, $F = \{2\}$, and $\delta$ given by the arcs $(1, b, 2)$, $(1, \epsilon, 2)$, $(1, a, 3)$, $(2, b, 3)$, $(3, b, 1)$, and $(3, a, 2)$ as shown in the following node-arc diagram:

(a) Calculate a DFA $M$ such that $L(M) = L(N)$.

(b) Calculate a regular expression $r$ such that $L(N) = L(r)$.
(3) (5 x 3 = 15 pts.) True/False.

Please write out the words true and/or false in full. No justifications are needed.

(a) If $A^* = A$, then the language $A$ includes the empty string.

(b) If $A$ and $B$ are regular languages recognized by 3-state DFAs, then $A \cap B$ can be recognized by a 6-state DFA.

(c) The empty relation on a nonempty set is transitive.

(d) The intersection of two non-regular languages is always non-regular.

(e) If there is a string $w$ such that no string $x$ in a regular language $A$ has $w$ as a substring, then every DFA $M$ such that $L(M) = A$ has a dead state.

(a) __________  (b) __________  (c) __________  (d) __________  (e) __________

(4) (7 pts.)

Let $L = \{x \in \Sigma^* : \text{both of the substrings } aa \text{ and } bb \text{ occur in } x, \text{ each at least once}\}$. Write a regular expression $r$ such that $L(r) = L$. (Hint: Break into two cases, one for $aa$ coming first, the other for $bb$ coming first.)
(5) (21 pts.)

Define $L = \{ x \in \{a, b\}^* : |x| \text{ is odd and the middle character of } x \text{ is a } 'b' \}$. For instance, $b$, $aba$, and $baabaaa$ belong to $L$, but $\epsilon$, $bbbb$, and $bbabb$ do not.

Prove using the Myhill-Nerode technique that $L$ is not a regular language. *Hint:* take $S = (aa)^*$ or $S = (bb)^*$. END OF EXAM.