Prelim I will be held in class period as advertised on Tuesday, March 8th. It will allow one notes sheet, front-and-back OK, and otherwise be closed-book, closed-notes. It covers all of chapter 1 except that the Myhill-Nerode Theorem replaces the “Pumping Lemma,” and Chapter 0 except that we haven’t had proofs by induction on homeworks yet. A copy of last year’s Prelim I will be posted on Tuesday for practice.

Next Tuesday’s lecture will do one more Myhill-Nerode example, summarize all that Chapter 1 told us about the class of regular languages, and transit to Chapter 2. So for next week please read all of section 2.1 “in one gulp”—well it’s only 10 pages but weirdly we will spend three weeks on them together with my notes on the course webpage about how proofs by induction relate to context-free grammars.

Assignment 4, due in hardcopy; please staple any multiple-sheet submission

(1) Write a regular expression $R$ over the alphabet $\{0, 1, 2\}$ such that $L(R)$ equals the language $A$ of numbers in base-3 that are congruent to either 0 or 3 modulo 4. Count the empty string as belonging to $L(R)$ (i.e., it counts as zero), and allow leading 0s in the numbers.

First design a DFA $M$ such that $L(M) = A$. Then show the conversion to a regular expression. For good measure, give a PD set of size 4 for this language based on your $M$.

As a hint for designing $M$, use four states for the possible congruences. The idea of an “even bank” and “odd bank” like in 1(g) of PS2 may help too. (24 pts.)

(2) Prove that the following two languages are non-regular via the Myhill-Nerode Theorem. Both use alphabet $\Sigma = \{0, 1\}$. Here $x^R$ stands for $x$ reversed, e.g. $0111^R = 1110$. The condition $x = x^R$ means that $x$ is a palindrome, but the members of $L_a$ are a little more restrictive, including 1001 but not 10001. (2 × 12 = 24 pts.)

(a) $L_a = \{xx^R : x \in \Sigma^*\}$.

(b) $L_b = \{x \in \Sigma^* : \#0(x) > \#1(x)\}$.

(3) Prove that the following language is nonregular via the Myhill-Nerode Theorem.

$L_3 = \{a^ib^jc^k : i = j \text{ or } i \neq k\}$.

(18 pts., for 66 total on the set)