CSE 396
Problem Set 4 Answer Key
Spring 2016

(1) \[ \Sigma = \{0, 1, 2, 3\}, \quad L = \{x \in \Sigma^* \mid \text{x mod 4} = 0 \text{ or } 3, \text{x in standard ternary}\} \]

OFA \( M \) has states
\[ Q = \{q_0, q_1, q_2, q_3\}, \]
one for each possible congruence mod 4.
\[ S = \{q_0, q_1\}, \quad F = \{q_0, q_3\}, \]
A PD set is \( S = \{0, 1, 2, 10\} \); see rest of key for proof.

Since there is only one accepting state besides start, we can save work by not adding a new final state with \( \varepsilon \)-arcs from the old ones like in the text. We can eliminate states \( q_2 \) and \( q_4 \) and get down to our "accelerated" two-state base case.

Elim \( q_2 \): Incoming

\[
\begin{array}{c|c|c|c|c}
\text{Update} & \text{State on} & \text{Self} & \text{Outgoing} & \text{State on} \\
\hline
T(0, 0) & 0 & 0 & 0 & 0 \\
T(0, 3) & 9 & 1 & 0 & 1 \\
T(3, 0) & 9 & 1 & 0 & 1 \\
T(3, 3) & 9 & 1 & 0 & 1 \\
\end{array}
\]

Elim \( q_4 \): Self

\[
\begin{array}{c|c|c|c|c}
\text{Update} & \text{State on} & \text{Self} & \text{Outgoing} & \text{State on} \\
\hline
T(0, 0)_{\text{new}} = T(0, 0)_{\text{old}} + T(0, 3)_{\text{old}} T(2, 2) \cdot T(2, 0) \\
& = 0 + 2 \cdot 9 &= 0 + 2 \cdot 0^* \cdot 1 = 20^* 1 \\
T(0, 3)_{\text{new}} = T(0, 3)_{\text{old}} + T(0, 2)_{\text{old}} T(2, 2) \cdot T(2, 3) \\
& = \emptyset + 2 \cdot 9 &= \emptyset + 2 \cdot 0^* \cdot 1 = 20^* 1 \\
T(3, 0)_{\text{new}} = T(3, 0)_{\text{old}} + T(3, 2)_{\text{old}} T(2, 2) \cdot T(2, 0) \\
& = 10^* 2 \\
T(3, 3)_{\text{new}} = 2 + 10^* 1. \text{ New GNFA at left.} \\
\end{array}
\]

\[
L(M) = L_{00} \cup L_{03}. \text{ This base case unions both languages.} \\
L_{00} = (2 + (2 \cdot 2^* \cdot 1) \cdot \varepsilon) = 0 + 2 \cdot 2^* + 12^* + 20^* 12^* + 20^* 12^* \\
L_{03} = L_{00} \varepsilon \\
L_{03} = (2 + (2 \cdot 2^* \cdot 1) \cdot \varepsilon) = (2 + (2 \cdot 2^* \cdot 1) \cdot \varepsilon) + \varepsilon \\
\text{This is fine to give as the answer.}
\]