Regular Expression Examples: You can follow this "naturally" without consciously applying the formal definitions.

I. How to represent Whole Numbers? \[ \mathbb{N} \]

Alphabet: \( \mathbb{D16} = \{ 0, 1, -9 \} \)

Idea 1: \( \mathbb{D16} \) As in decimal, the * means "zero or more".

Problem: allows the empty symbol \( \epsilon \). "Invisible Zero".

Idea 2: \( \mathbb{D16}^+ = \) Superscript + means "one or more".

Second Issue: Allows redundant codes for integers with leading 0s.

\( 007 = 7 = 0000000007 \)

Idea 3: Require the first digit to be 1 until \( \mathbb{D16} \):

\( \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \} \cdot \mathbb{D16}^+ \)

or \( \text{UNIX:} \) shaker \( [1-9] \) followed by \( 0, 1 \)

Problem: 0 has disappeared.

Idea 4: Union it as a special case:

\( \mathbb{N}_{\text{with}} = [1-9] \cdot \mathbb{D16}^+ \cup 0 \)

Now since we allow negative numbers, we might use an (option-1) _sign_.

\( \)
Notation for "BNF Grammar"  

UNIX/Linux  

Regex/Text  

\([-\text{U} \text{E}] ([-1-9] \text{DIG}^* \text{ u 0}) \text{ allows } -0\]

\([-\text{U} \text{E}] [-1-9] \text{DIG}^* \text{ u 0} \text{ does not allow } -0\]

\([-\text{U} +\text{U} \text{E}] [-1-9] \text{DIG}^* \text{ u 0} \text{ allows optional + sign too.}\]

III: Now how about Floating Point numbers?

\[\text{INT} \cdot \text{DIG}^+\]  

New leading 0s are needed in the floating point, but trailing 0s cause rounding. OK.

Issue: can't write .5  

must write 0.5. Bug or feature?

If you write \([-\text{U} +\text{U} \text{E}] \text{DIG}^* \cdot \text{DIG}^*\] you can do .5,  

but then also get .5\text{u} or .5\text{i}?”  

Of course we can't allow the latter, so let's  

\[\text{INT} \cdot \text{DIG}^* \cdot \text{DIG}^* \text{ or } (-\text{U} +\text{U} \text{E}) \text{DIG}^* \cdot \text{DIG}^* \]
Formal definition and notation:

\[ A \circ B = \{ x \cdot y : x \in A \land y \in B \} \]

Contrast with Cartesian Product:

\[ A \times B = \{ (x, y) : x \in A \land y \in B \} \]

\[ A = \{ a, ab \} \quad A \circ B = \{ a \cdot b, a \cdot bb, ab \cdot b, ab \cdot bb \} \]
\[ B = \{ b, bb \} \quad = \{ ab, ab \cdot b, ab \cdot bb \} \quad \text{size 3} \]
\[ A \times B = \{ (a, b), (a, bb), (ab, b), (ab, bb) \} \]
\[ \text{almoins } |A \times B| = |A| \cdot |B|. \quad \text{here, } = 4 \]

\[ A \circ A = \{ x \circ y : x \in A \land y \in A \} \]
\[ \neq \{ x \cdot x : x \in A \} \quad \text{which would be } \subseteq \text{ the } D\text{-algebra (not used)} \]

\[ A^1 = A, \quad A^2 = A \circ A, \quad A^3 = A \circ A \circ A. \quad \text{(hence, } 0 \text{ is additive)} \]

Rule for Exponent:

\[ A^a \cdot A^r = A^{a+r} \quad \text{like } 0^0 = 1. \]

Needs \( A^0 = \{ \text{sets} \} \quad \text{not } \emptyset. \quad \text{Even } \emptyset^0 = \{ \emptyset \} \quad \text{not } \emptyset! \]