What is $0^0$? Why do people say $0^0 = 1$?

The reason we care: For any language $A$ over an alphabet $\Sigma$, and $i \in \mathbb{N}$, define $A^i = \{ \text{concatenations of } i \text{ strings} \}$ such that each string belongs to $A^i$.

For instance, if $A = \{ab, aba, ba\}$ and $i = 2$, then $A^2 = \{abab, ababa, abba, abaab, abacaba, baab, baba, baba\}$.

- Note that we did not list "ababa" twice, even though it came up twice, as ab·aba and later as aba·ba. This is because $A^2$ is a set, not a list.
- Note that the strings being concatenated don't have to be different: we included ab·ab, aba·aba, and ba·ba.
- Note that $A^2 \neq \Sigma \times \times : x \in A^2$. The latter is just $3(abab, ababa, baba)$.
- This is a case of the more general definition of concatenation of languages which will be given (next week) in lectures. For languages $A$ and $B$,

$$A \cdot B = \{ xy : x \in A \text{ and } y \in B \}.$$ (Jan 25 or 29)

Then $A^2$ equals $A \cdot A$, i.e. the case $B = A$ here. This accords with how powering relates to multiplication in numerical math, but now with strings and symbols. How far does the analogy go?

Well, $A \cdot \emptyset = \{ xy : x \in A \text{ and } y \in \emptyset \} = \emptyset \cdot \emptyset = \emptyset$. Like $A \cdot 0 = 0$

$$A \cdot \Sigma^i = \{ xy : x \in A \text{ and } y \in \Sigma^i \} = \{ xy : x \in A \text{ and } y = \Sigma^i \} = \{ x \cdot \Sigma : x \in A \} = A.$$ Like $A \cdot 1 = A$. 