So this exemplifies what last Wednesday’s lecture said about \( \emptyset \) being analogous to zero, and \( \mathbb{E} \) behaving like \( 1 \).
But now we want to push the analogy to obey this law of exponents:

\[
\text{Numbers: } \quad a^2 \cdot a^3 = a^{2+3} \quad \text{Works when } i=0 \text{ if } a^0=1
\]
\[
\text{Strings: } \quad x^i \cdot x^j = x^{i+j} \quad \text{OK for } i=0 \text{ if } x^0 = \epsilon
\]
\[
\text{Languages: } \quad A^i \cdot A^j = A^{i+j} \quad \text{OK for } i=0 \text{ if } A^0 = \mathbb{E} \mathbb{E} \mathbb{E}.
\]

So we make a convention: For every language \( A \), \( A^0 = \mathbb{E} \mathbb{E} \mathbb{E} \).
And we declare this true even for \( A = \emptyset \): \( \emptyset^0 = \mathbb{E} \mathbb{E} \mathbb{E} \). But why??

How can we get something out of nothing? (Lecture said \( \mathbb{E} \mathbb{E} \mathbb{E} \) is something)

*[Famous Buddhist "Koan" riddle:]*

What is the sound of one hand clapping?
I don’t have an answer. But for zero

\[\text{hands I do: The sound of 0 hands clapping is the empty sound, } \epsilon.\]
Then \( \emptyset^0 = \mathbb{E} \mathbb{E} \mathbb{E} \) is "merely" the further step of saying this is true even when there are no hands.

We would feel more comfortable about this if in math, \( 0^0 = 1 \).
Here’s the argument: In Discrete Math, for any sets \( P \) and \( Q \), \( Q^P \) stands for the set of functions \( f: P \to Q \). And by rule, \( |Q^P| = |Q|^{|P|} \).

For example, let \( P = \{0,1,2,3,4\} \) and \( Q = \{0,1,2\} \). Then a function \( f: P \to Q \) is the same as a binary string of length 5. E.g. the string 00110 is the function \( f(0) = 0, f(1) = 2, f(2) = 1, f(3) = 1, f(4) = 0 \). There are \( 2^5 = 32 \) such binary strings, and \( |Q^P| = 181^{10^1} = 2^5 \) as needed.

Hence, \( \emptyset^0 \) stands for the set of 32 functions \( f: \emptyset \to \emptyset \). Are there any?
Yes! \( \emptyset \) is a function from \( \emptyset \to \emptyset \). It’s the only one, so \( 0^0 = |\emptyset|^{|\emptyset|} = 1 \!\)