Reading:

We will finish Chapter 1 on Tuesday and transit to Chapter 2. So please read section 2.1 in one gulp, even though it will take us over spring break. This is the last material in the domain of Prelim I, which covers only through Chapter 1 and the assigned readings. Again, Prelim I is on Tuesday, March 14, in class period.

Assignment 4, due in hardcopy and in class Thu. 3/9* (*except R4 may submit Fri. 3/10 before 2pm) Please staple any multiple-sheet submission.

And: please write your name, Student ID#, and recitation attended atop your HW.

(1) Convert the following finite automata $N_1$ and $N_2$ into regular expressions $r_1$ and $r_2$ such that $L(r_1) = L(N_1)$ and $L(r_2) = L(N_2)$. No comments are needed on the final products, but you should show your work in the conversion process clearly—in particular noting any “reasonable shortcuts.” ($2 \times 18 = 36$ pts.)

\[
\begin{array}{c}
N_1 = (Q_1, \Sigma, \delta_1, s_1, F_1) \text{ with } Q_1 = \{1, 2, 3, 4\}, \Sigma = \{a, b\}, s_1 = 1, F_1 = \{1, 2\}, \text{ and } \\
\delta_1 = \{(1, a, 2), (1, b, 3), (2, a, 3), (2, b, 1), (3, a, 4), (3, b, 1), (4, a, 3)\}.
\end{array}
\]

\[
\begin{array}{c}
N_2 = (Q_2, \Sigma, \delta_2, s_2, F_2) \text{ with } Q_2 = \{1, 2, 3, 4\}, \Sigma = \{a, b\}, s_2 = 1, F_2 = \{2, 4\}, \text{ and } \\
\delta_2 = \{(1, a, 2), (1, b, 4), (2, a, 3), (2, b, 4), (3, a, 1), (3, b, 3), (4, a, 1), (4, b, 3), (4, \epsilon, 2)\}. \text{ Note the last } \epsilon\text{-arc.}
\end{array}
\]

(2) Given any strings $u$ and $x$ (with $u \neq \epsilon$), define $\#u(x)$ to be the number of times $u$ occurs as a substring of $x$. For example, if $u = 010$ and $x = 01010$ then $\#u(x) = 2$ because two occurrences are counted even though they overlap. Using the Myhill-Nerode technique (not the Pumping Lemma), prove that two the following languages are not regular—and give a regular expression for the one that is regular. ($3 \times 12 = 36$ pts.)

(a) $L_1 = \{x \in \{0, 1\}^* : \#00(x) > \#11(x)\}$.

(b) $L_2 = \{x \in \{0, 1\}^* : \#01(x) > \#10(x)\}$.

(c) $L_3 = \{x0y : \#0(x) = \#1(y)\}$. 