This will be treated as extra credit—that is, the denominator for homework points will stay at 528 from assignments 1–9. The points are still of course converted into course points using the same \( \frac{200}{528} = \frac{25}{66} \) multiplier.

(1) Lipton-Regan text, exercises 3.12 and 3.13 (24 pts. total)

(2) Lipton-Regan text, exercises 4.11 and 4.12, OK to skip the “argue generally” part of the latter. (Note the references to problem 4.8, and 4.10; 18 pts. total)

(3) Lipton-Regan text, exercises 7.4 and 7.7. Answer the last part of 7.7 taking problem 7.5 as fact. (Note how this connects to problem 4.8 again. 24 pts. total)

(4) Consider the \( 2 \times 2 \) Hadamard matrix together with its three rotations—for fun, let’s call them all the “Damhard” matrices:

\[
H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}; \quad H_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}; \quad H_3 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}; \quad H_4 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.
\]

Begin a quantum circuit \( C \) with 2 qubits by placing an Hadamard gate on line 1 followed by CNOT with control on 1 and target on line 2. As shown in lecture from ch. 7, on input \( e_{00} \) this creates an entangled pair of qubits—say the one on line 1 is owned by “Alice” while the second qubit is sent to “Bob.” Now add to line 1 a “black box” in which Alice has placed one of the four Damhard matrices. Your task is to finish \( C \) with some gates so that by measuring both qubits, Bob can learn exactly which one Alice used.

For a footnote relating to lecture, it is not possible to learn exactly in the case of the four matrices in Deutsch’s problem, even if we added a third qubit to the circuit that could be entangled with the others and kept by “Bob.” The reason is that those four matrices are not linearly independent: \( U_I + U_X = U_T + U_f \). Thus if you have any vector \( u \), the four vectors \( v_1 = U_I u, v_2 = U_X u, v_3 = U_T u, \) and \( v_4 = U_f u \) resulting from them are linearly dependent. Hence the vectors \( w_1, w_2, w_3, w_4 \) you would get from later stages of the circuit are also linearly dependent. This means in particular that their nonzero entries must overlap in some indices, and any such overlap prevents 100% certainty that a single measurement will distinguish them.

However, the Damhard matrices are linearly independent. For a help in building the rest of \( C \), try multiplying each of \( H_2, H_3, H_4 \) by \( H \). Then relate the results to the four “Pauli matrices” which the text uses for its demonstration of “superdense coding” in section 8.2. The final helpful fact from lecture is that a scalar unit constant like \(-1\) or \( i \) never changes any measurement, so you can disregard it. (30 pts. total, for 96 on the set.)