Lectures and Reading. Next week will continue reductions showing NP-hardness from the ALR notes and Arora-Barak. Friday may get to diagonalization from Arora-Barak chapter 3.

(1) A short-essay topic

We refer to the proof of the Cook-Levin Theorem that was given in class. Suppose we were to negate every variable in every block of three clauses for incoming wires $u, v$ and outgoing wire $w$ from a gate $g$, so that it now reads:

$$(\bar{u} \lor \bar{w}) \land (\bar{v} \lor \bar{w}) \land (u \lor v \lor w).$$

But suppose we still require that the output wire $w_0$ outputs 1 for acceptance. Is the construction still correct—or correctable? And please show exactly what would happen if we were to use the other universal gate, NOR, in place of NAND in the construction of $\phi_n$ in the proof. (18 pts.)

(2) Show that the problem of deciding whether an undirected graph is connected belongs to polynomial time. Sketch an algorithm in pseudocode and estimate its asymptotic running time in terms of the number $n$ of vertices and number $m$ of edges. (Note that this could be part of a witness predicate for the “Unbreakable” problem on Assignment 5. 18 pts.)

(3) Choosing the New Administration.

The President-elect needs to fill a fairly large number of federal positions—many more besides the Cabinet—from an even larger number $n$ of qualified people. He/she also listens to various advisors and special-interest groups, who submit a large number $m$ of positive and negative recommendations. A positive recommendation lists 1 or 2 or 3 or more people and says, “choose at least one of these guys.” A negative recommendation gives a similar list but says, “Don’t choose all of these guys—that would be favoring this group too much.” The recommendations don’t necessarily specify a particular federal position—they only talk about whether particular people $x, y$, and/or $z$ should belong to the Administration in general.

Consider the task of choosing whether each person should belong to the Administration. Can this be done in a way that meets every recommendation? Prove that the decision problem framed by this question is NP-complete, preferably by reduction from (3)SAT and in any event by showing how this problem is “SAT-like.” (Thus in a formal, computational sense it is hard to form a government. 21 pts., for 57 on the set)