Lectures and Reading. Friday (Nov. 4) will cover some other things out of Arora-Barak chapter 2 and then may do a brief treatment of the DSPACE hierarchy Theorem going back to Chapter 27 of the ALR notes (which were given out back in Week 1). Note that Arora-Barak mentions this at the top of page 75 in its Chapter 4 but says the proof is “completely analogous to that of the Time Hierarchy Theorem in Chapter 3”—instead I may treat it as a warmup for covering the THT in full out of the same ALR pages and Arora-Barak chapter 3 next Monday. Then we will transit to Chapter 4 and the parallel material in ALR chapters 27 and 28, which will occupy us most of this month.

The following is due next Friday to allow for Election Night hangover and more time on longer problems.

(1) (36 pts. total)

Prove that the following decision problem is NP-complete.

Two-Coloring With Faults

Instance: An undirected graph \( G \), an integer \( k \geq 1 \).

Question: Can \( G \) be colored with 2 colors so that at most \( k \) edges have both nodes of the same color?

Note: You may use any variant of (3)SAT that was covered in the course, including the “2-3” hybrid used in the Cook-Levin proof given in lecture, or forms such as “not-all-equal 3SAT.” Or you may use a problem proved NP-complete on homeworks. But you are not allowed to use “any” problem from an Internet or otherwise unofficial source—and it is strongly recommended that you use the “ladder/clause-gadget” architecture covered in the course for reductions from (3)SAT.

(2) (36 pts. total)

Arora-Barak, chapter 2, exercise 2.17, both Exactly One 3SAT and Subset Sum. Needless to say, it is forbidden to look these up on the Internet.