## Reconsideration on Non-Linear Base Orderings

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**Abstract.** Reconsideration is a belief change operation that re-optimizes a finite belief base following a series of belief change operations—provided all base beliefs have a linear credibility ordering. This paper shows that linearity is not *required* for reconsideration to improve and possibly optimize a belief base.

Keywords. Base belief change, knowledge base optimization, reconsideration

Reconsideration, as defined in [2] (and discussed in [3] in these proceedings), reoptimizes a finite belief base in an implemented system following a series of belief change operations, provided the base beliefs have a linear credibility ordering; but ordering *all* the base beliefs in a knowledge system is impractical. This paper shows that linearity is not *required* for reconsideration to improve and possibly optimize a belief base.<sup>1</sup>

A belief *base*, for implementation purposes, is a finite set of core (or base) beliefs that are input to the system. Any implemented system that can perform expansion (adding a new belief to the base) and consolidation (removing beliefs from the base to restore consistency [1]) can perform reconsideration.

We define the minimally inconsistent subsets of a base *B* as *NAND-sets*; a NANDset that is a subset of the current base is called *active* and makes that base inconsistent. Consolidation of *B* (written *B*!) uses a decision function to select the base beliefs (called culprits) to be removed (unasserted). In addition to (and assumed to be consistent with) any pre-existing credibility ordering, the selected culprits are considered strictly weaker than other members of their NAND-sets that were *not* removed. The system must store all base beliefs (asserted and unasserted) in a set called  $B^{\cup}$  in order to perform reconsideration, which is the consolidation of all base beliefs ( $B^{\cup}$ !) and is independent of the current *B*. An unasserted culprit is *JustifiedOut* if its return raises an inconsistency that can be resolved only by removing either that culprit or some *stronger* belief.

We define an optimal base by assuming a consistent base is preferred over any of its proper subsets and a belief p is preferred over multiple beliefs (e.g., q, v) that are strictly weaker than  $p: p \succ q; p \succ v; \therefore \{p\} \succ \{q, v\}$ . If the pre-order defines a *least element* for all NAND-sets, the following algorithm yields an optimal base. Let B be the set of all non-culprit base beliefs in  $B^{\cup}$ . For each culprit p (in non-increasing order of credibility): if p is *not* JustifiedOut, reset  $B \leftarrow B \cup p$ . After each pass through the for-loop:

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<sup>&</sup>lt;sup>1</sup>See [2] (or [3]) for a detailed (or brief) discussion of the benefits of reconsideration.

Table showing a base, B, revised by $\neg a$ (.95), then revised by a (.98), and then after Reconsideration is performed.							
Columns show different adjustment strategies producing varied results for revision and reconsideration.							
Belief Base	Degree	Standard	Maxi-adjustment	Hybrid	Global	Linear	Quick
В	.95	$a \lor b$					
	.90	$a \lor f$					
	.40	$a \lor d$ , $\neg b \lor \neg d$ ,	$a \lor d, \neg b \lor \neg d,$	$a \lor d, \neg b \lor \neg d,$	$a \lor d$ , $\neg b \lor \neg d$ ,	$a \lor d, \neg b \lor \neg d,$	$a \lor d, \neg b \lor \neg d,$
		d, e, f					
	.20	$\neg g \vee \neg b, \neg d \vee g$	$\neg g \vee \neg b, \neg d \vee g$	$\neg g \vee \neg b, \neg d \vee g$	$\neg g \vee \neg b, \neg d \vee g$	$\neg g \vee \neg b, \neg d \vee g$	$\neg g \vee \neg b, \neg d \vee g$
$(B + \neg a)!$	.95	$\neg a, a \lor b$					
	.90	$a \lor f$					
	.40	f	e, f	$\neg b \lor \neg d, e, f$	e, f		d, e, f
	.20		$\neg g \vee \neg b, \neg d \vee g$	$\neg g \vee \neg b, \neg d \vee g$		$\neg g \vee \neg b, \neg d \vee g$	
$((B + \neg a)! + a)!$	.98	a	a	a	a	a	a
	.95	$a \lor b$	$a \lor b$	$a \lor b$	$a \lor b$		$a \lor b$
	.90	$a \lor f$					
	.40		e, f	$\neg b \lor \neg d, e, f$	e, f		d, e, f
	.20		$\neg g \vee \neg b, \neg d \vee g$	$\neg g \vee \neg b, \neg d \vee g$		$\neg g \vee \neg b, \neg d \vee g$	
$((B + \neg a) + a)!$	.98	a (improved)	a (optimal)	a (optimal)	a (unchanged)	a (improved)	a (optimal)
	.95	$a \lor b$	$a \lor b$	$a \lor b$	$a \lor b$		$a \lor b$
	.90	$a \lor f$					
Reconsideration	.40	$a \lor d$	$a \lor d, \neg b \lor \neg d,$	$a \lor d, \neg b \lor \neg d,$	e, f	$a \lor d, \neg b \lor \neg d,$	$a \lor d, \neg b \lor \neg d,$
			d, e, f	d, e, f		d, e, f	d, e, f
	.20		$\neg g \lor \neg b, \neg d \lor g$	$\neg g \lor \neg b, \neg d \lor g$		$\neg g \lor \neg b, \neg d \lor g$	$\neg g \lor \neg b, \neg d \lor g$

 Table 1. This table shows revision and reconsideration on a total pre-order of beliefs using six different adjustment strategies (as implemented in SATEN[4]). For a full discussion, cf. [2].

- 1. if q is a culprit and  $q \succ p$ , q was processed during an earlier pass;
- 2. all NAND-sets with *p* as a least element are *not* active and will remain so through the end of the algorithm;
- 3. if p is JustifiedOut, it will remain so through the end of the algorithm.

When the loop exits, we know that:

- all unasserted culprits are JustifiedOut;
- the resultant base, *B*, is consistent (no NAND-set is active);
- the resultant base, B, is optimal  $(\forall B' \subseteq B^{\cup} : B' \neq B \Rightarrow B \succ B')$ .

When the minimal beliefs of a NAND-set number more than one, base optimality is harder to define, but reconsideration can still help improve a base (possibly to a clearly optimal state). Table 1 shows reconsideration on a total pre-order for six different decision functions implemented in SATEN [4]. Five bases improved—three to optimal.

Systems with non-linear credibility orderings can benefit from implementing reconsideration. We have implemented an anytime, interleavable algorithm for reconsideration in an existing reasoning system (cf. [2]).

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