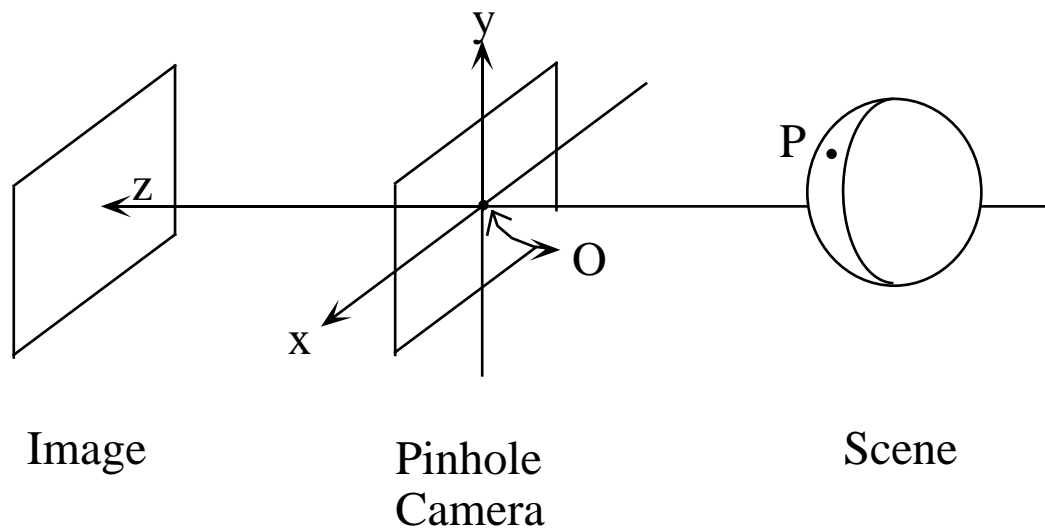Projections of 3-D Scenes onto a 2-D Plane

We draw a cube: how do we come up with the projection?
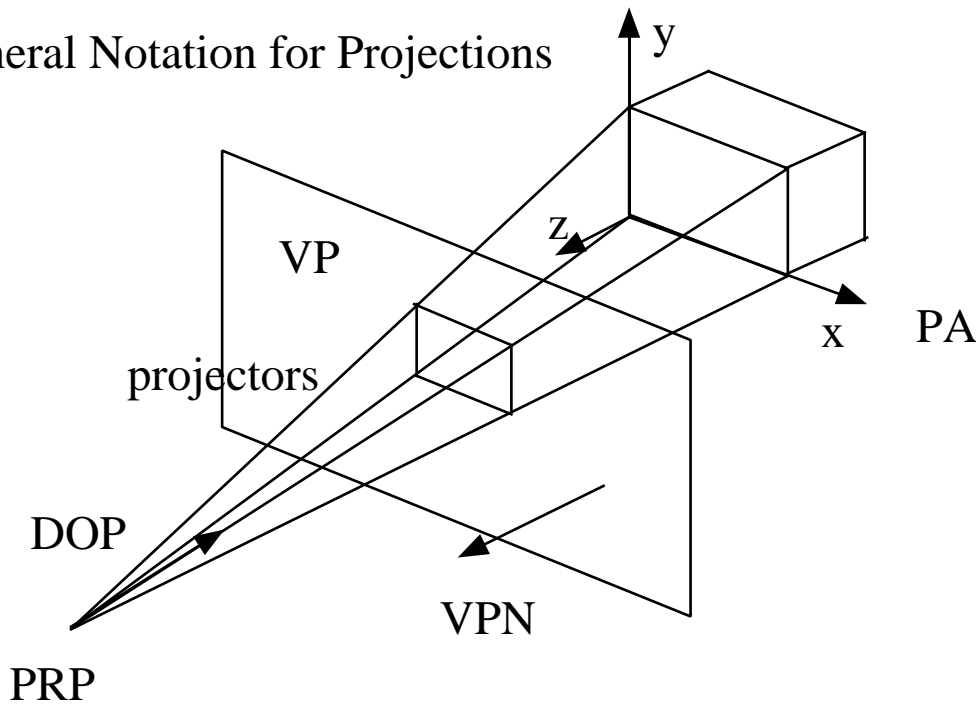
We photograph a scene: how does the light reflected from
the scene via the lens form the projection?
(see figure)

Image   Pinhole   Scene
                         Camera

Ideal Camera: Pinhole Camera

Where does point P in the scene project to in the image?

General Notation for Projections
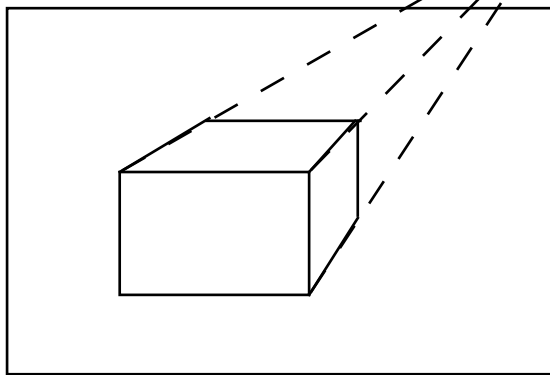

PA	principle axes of the scene
VP	view plane (projection plane)
VPN	view plane normal (projection plane normal)
PRP	projection reference point (center of projection)
DOP	direction of projection

Perspective Projections

finite PRP

VP

z

y

x   PA

projectors

DOP

VPN

PRP
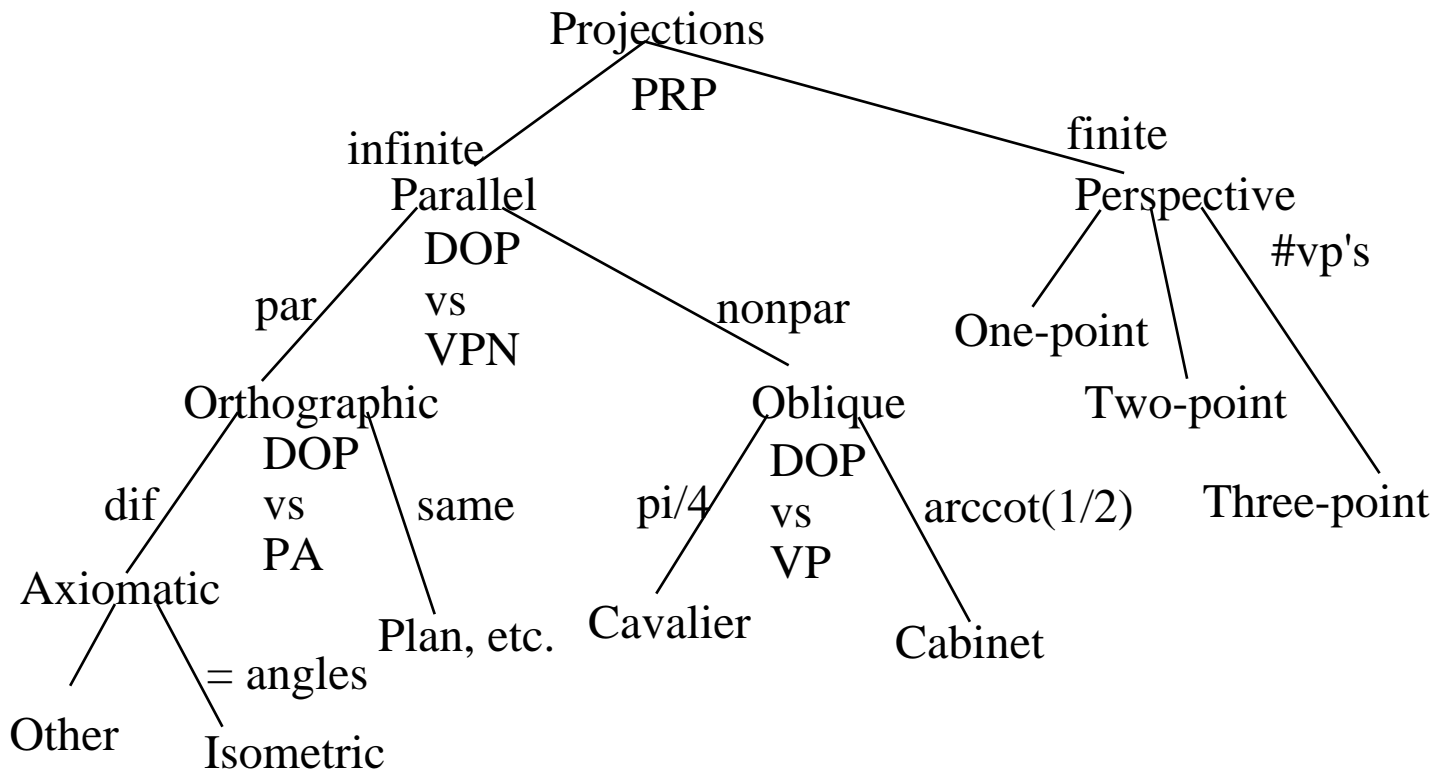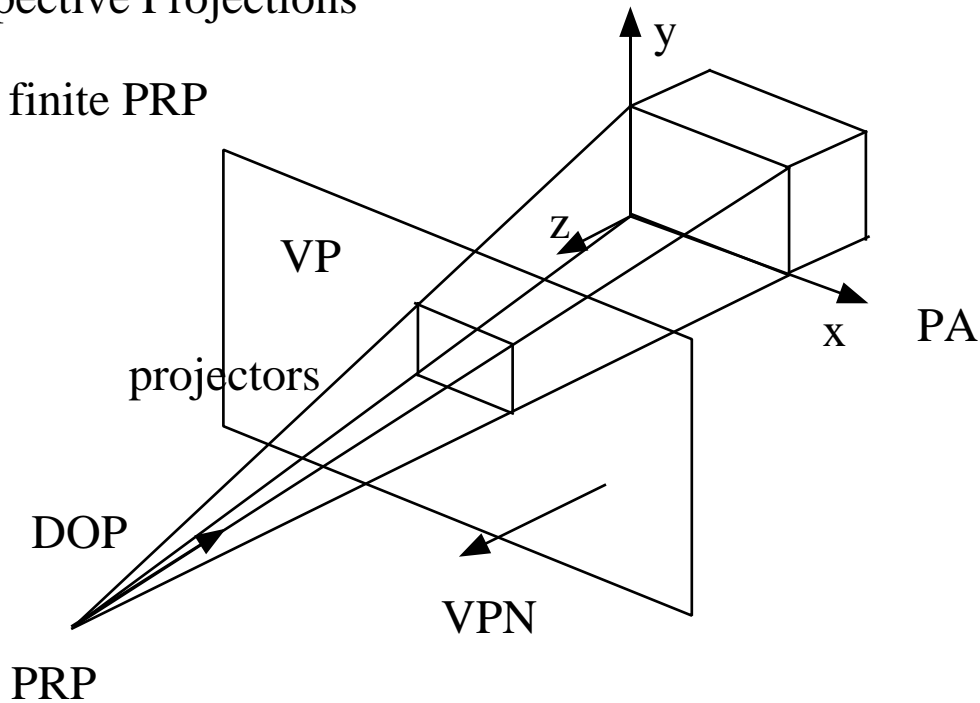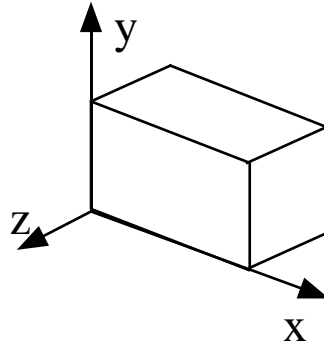
What does projection look like?   vanishing point

How get more than one vanishing point?

It's a function of the relation between the principle axes and
the viewing plane.

Number of vanishing points = number of principle axes cut by
viewing plane

How get two-point perspective?

Usually cut x and z axes

Where place view plane?

(see figure for projection)

Three-point perspective

Cut all three principle axes

Not used much, as doesn't increase realism

Relation between SLR cameras and perspective projections

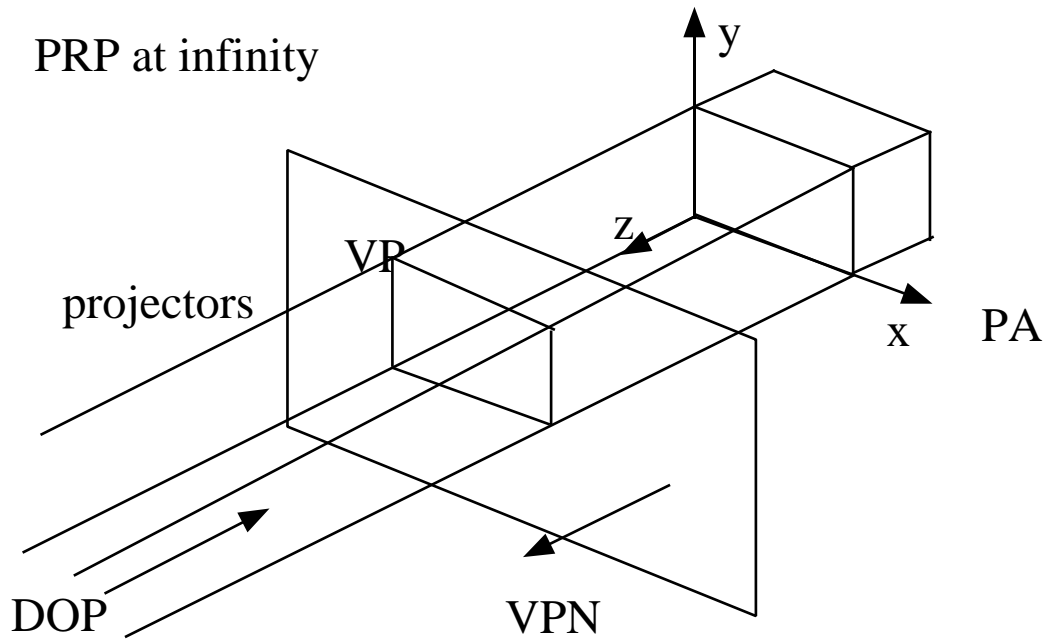How does photographer determine which projection?

How to get one-point perspective image of class?
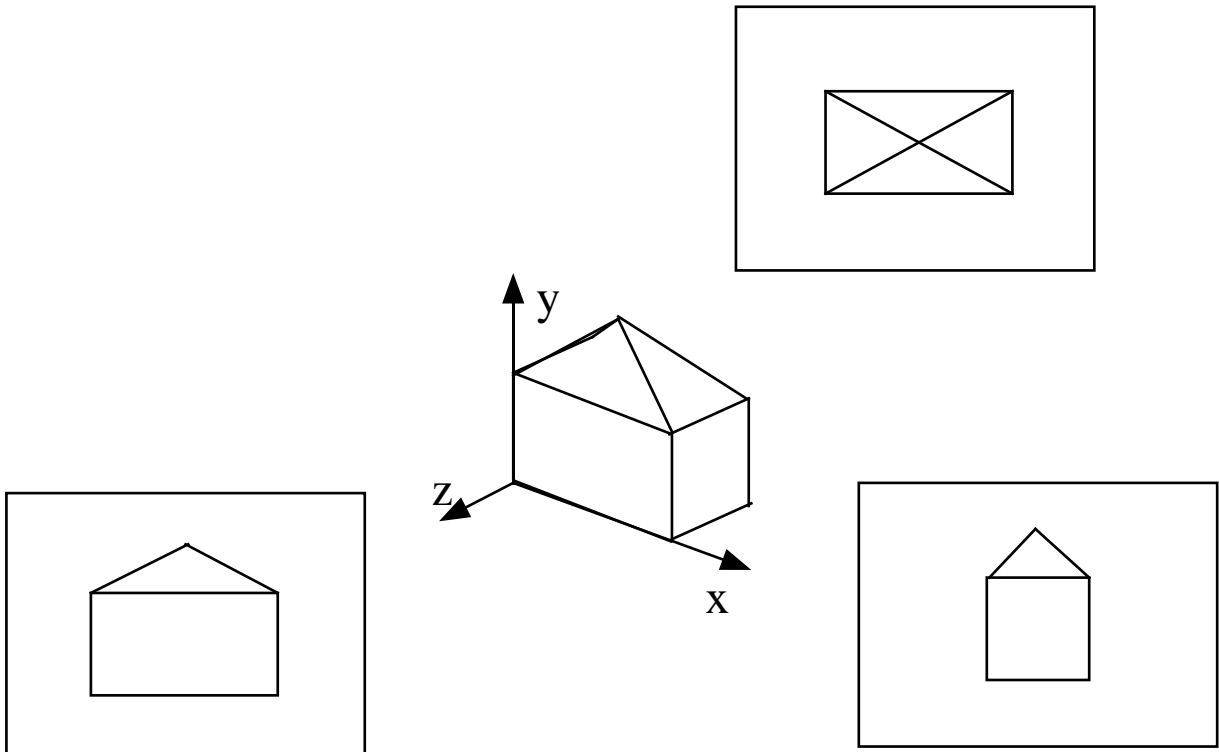two-point?
three-point?

We view (our visual systems) with perspective projections
distant objects smaller
perspective foreshortening

Parallel Projections

PRP at infinity

Which parallel projection does this depict?

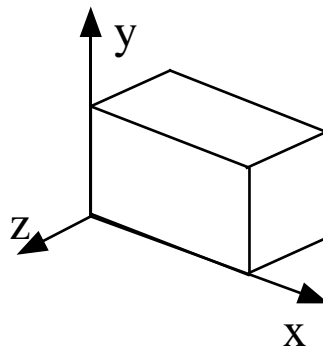Plan, front elevation, side elevation

Axiomatic Projections

     DOP not aligned with a principle axis

     Isometric Projection

         DOP makes equal angles with each principle axis
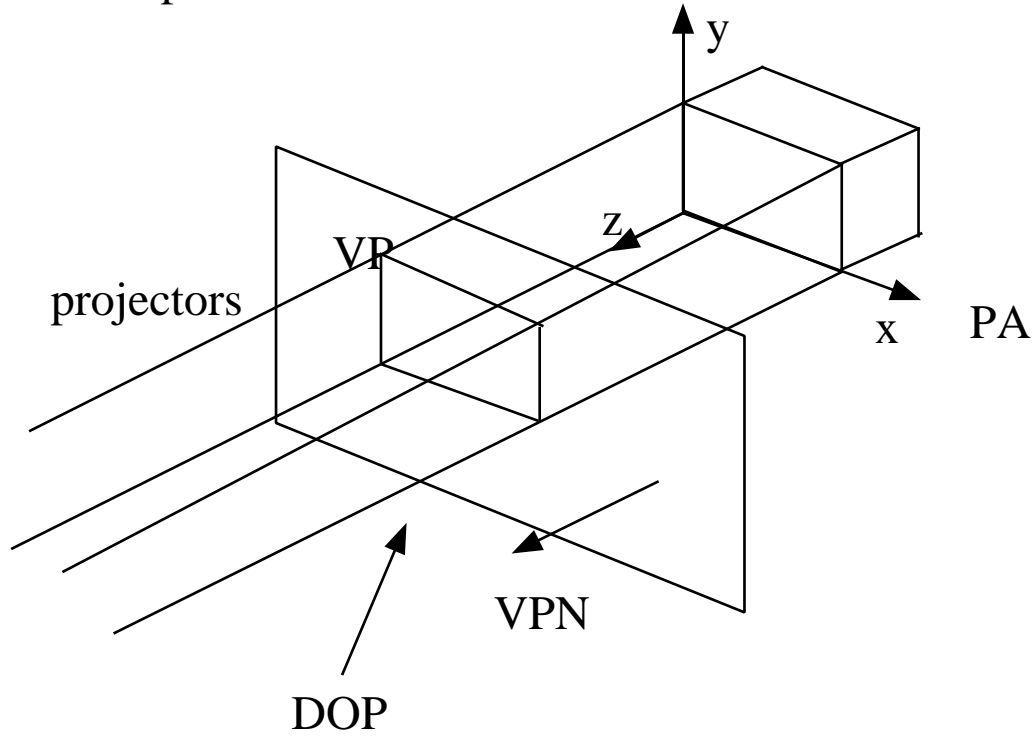

     What would DOP be here?

     What would the projection look like?

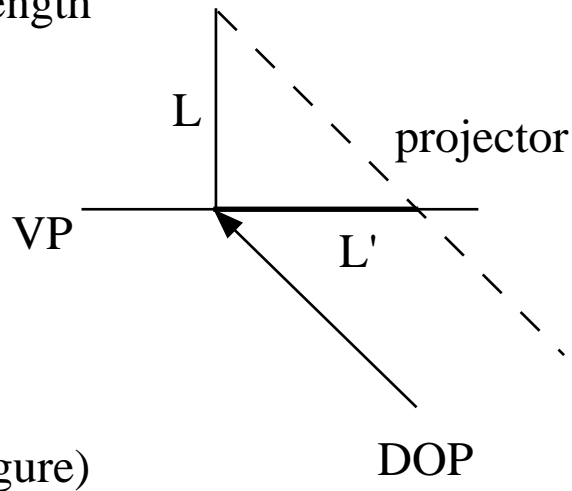Oblique Projections

DOP is not parallel to VPN

Cavalier Projection

DOP forms 45 degree angle with VP

Does this fully specify DOP?

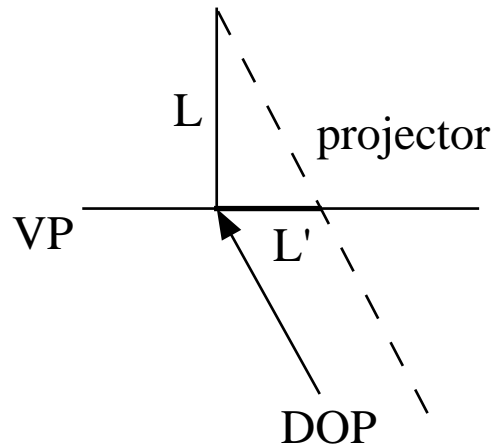All lines perpendicular to the VP are projected with no change
in length

(see figure)

Cabinet Projections

DOP makes an angle of arccot(1/2) with VP

Lines perpendicular to the VP are projected at half length

angle ~= 63.4 degrees

(see figure)

Specification of an Arbitrary 3-D View

Need not only projection, but also the view volume

(see figure)

Must specify window on the VP

Must specify front and back (hither and yon) planes

Commonly use a rectangular window
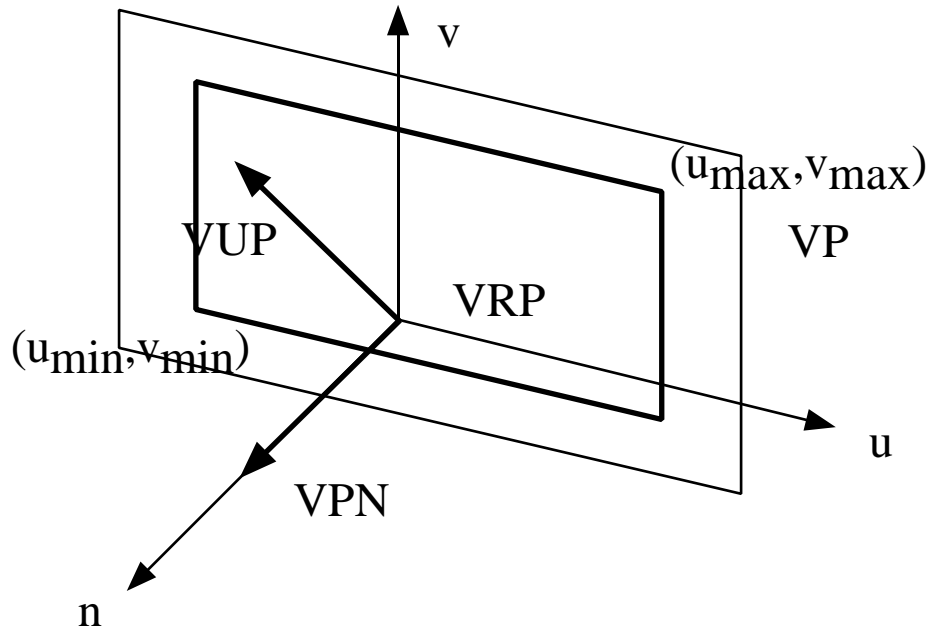
What window shape used by SLR camera?
What shape view volume?
What window shape used by eye?
What shape view volume?

Assume rectangular window

What do we need to specify for rectangular window?

First form viewing reference coordinate (VRC) system

Assign some point in the plane to be its origin
Call it the viewing reference point (VRP)
VPN and origin specify the n axis
VUP (view-up vector) defines v axis, as the
projection of VUP onto VP
(projection in VPN-VUP plane)
u axis is perpendicular to v and n axes

Now can specify window

v and u axes are the principle axes of the window
define window by (u_{max}, v_{max}) and (u_{min}, v_{min})

How get a rotated window?

Note center of window (CW) not necessarily at VRP

How to specify the view volume?

For perspective projection use window, PRP, and hither and yon planes

v

(u_{max}, v_{max})

VUP

VP

VRP

(u_{min}, v_{min})

u

VPN

n

PRP

For parallel projection, use window and hither and yon planes

(see figure)