# Three Dimensional Representations

- Quadric Surfaces
- Superquadrics
- Sweep Representations
- Constructive Solid Geometry
- Octrees

## Quadric Surfaces

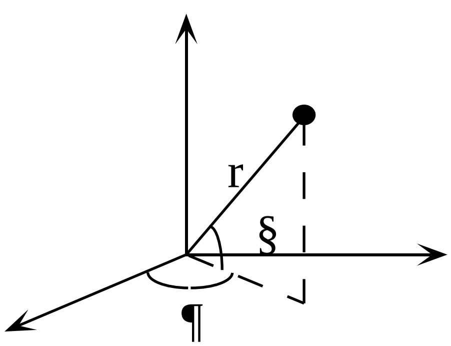
Second degree equations - quadratics

Sphere

nonparametric: $x^2 + y^2 + z^2 = r^2$

parametric: use latitude (§) and longitude (¶)

x = r cos § cos ¶          -¹/2 ² § ² ¹/2
y = r cos § sin ¶          -¹ ² ¶ ² ¹
z = r sin §




use 0 ² u,v ² 1
§ = ¹u
¶ = 2¹v

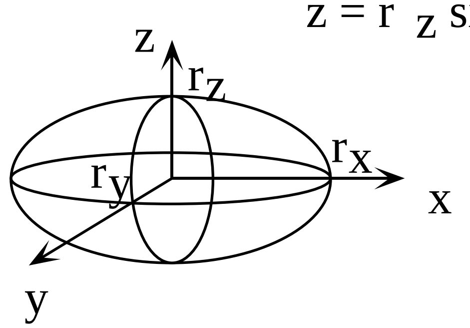
## Ellipsoid

nonparametric: $(x/r_x)^2 + (y/r_y)^2 + (z/r_z)^2 = 1$

parametric: $x = r_x$ cos § cos ¶ -1/2 2 § 2 1/2

$y = r_y$ cos § sin ¶ -1 2 ¶ 2 1

$z = r_z$ sin §




## Torus

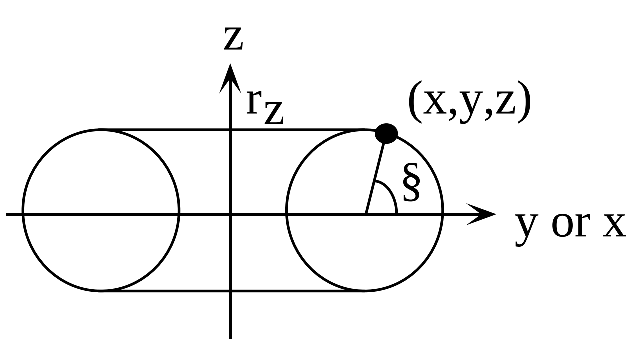
Rotate circle (or other conic) in yz plane about z axis

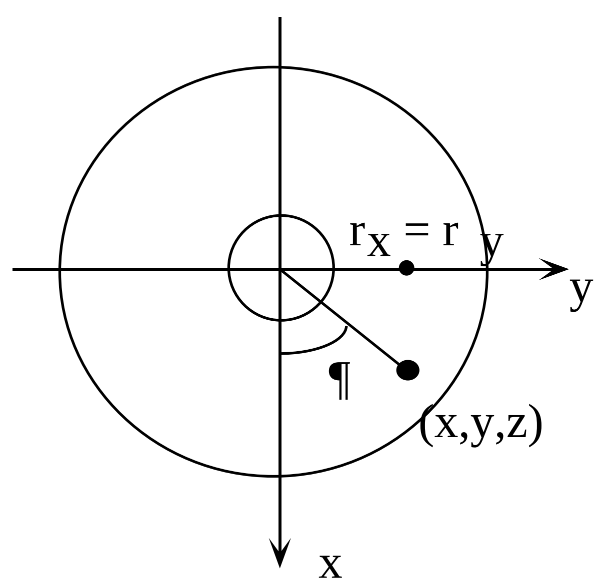
Parametric:

$x = r_x$ ( r + cos § ) cos ¶ -1 2 § 2 1

$y = r_y$ ( r + cos § ) sin ¶ -1 2 ¶ 2 1

$z = r_z$ sin §



# Superquadrics

Generalize quadrics by adding extra parameters

Number of extra parameters equals dimension of object

one extra for curves

two extra for surfaces

## Superellipse

parametric: $x = r_x \cos^{s}$ ¶ -1 2 ¶ 2 1

$y = r_y \sin^{s}$ ¶

(see figure)

## Superellipsoid

parametric: $x = r_x \cos^{s1}$ § $\cos^{s2}$ ¶ -1/2 2 § 2 1/2

$y = r_y \cos^{s1}$ § $\sin^{s2}$ ¶ -1 2 ¶ 2 1

$z = r_z \sin^{s1}$ §

(see figure)

How to define shapes for manufacture?

- Consider the process used to form the shape
  - Surfaces of revolution
    - wood turned on a lathe
    - pot formed on potters wheel
  - Extrusion
    - spaghetti
    - plastic siding and gutters
  - Rolling, forming, molding
    - clay finger pots
    - pie crusts
  - Sawing, flat grinding
    - construction 2x4's
- Develop representation appropriate for the process
  - Examples:
    - sawing and flat grinding yields polygons
    - surface of revolution yields surfaces represented by sweep representations

Computer graphics systems for design should allow user to easily create appropriate surfaces

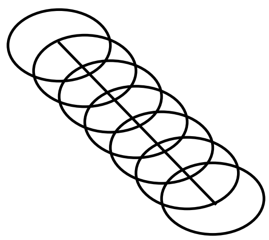
# Sweep Representations

Define by: two-dimensional shape
sweep that moves the shape through a region of space

## Translational sweep

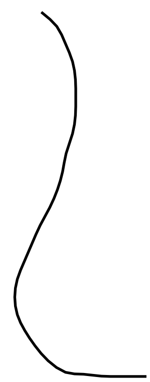
example:

Sweep an ellipse along a line



## Rotational Sweep

example:



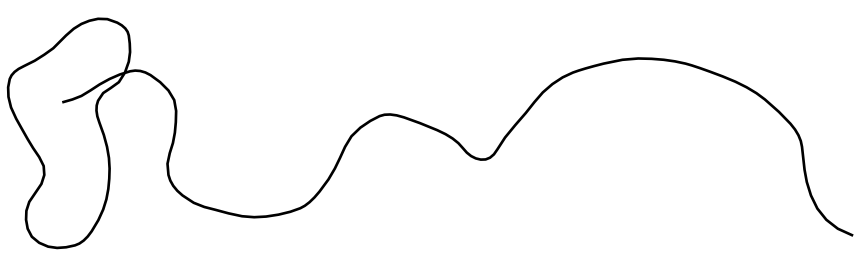
How get a torus?

## Arbitrary Sweep

example:

Define cross section as a closed curve represented as a B-spline

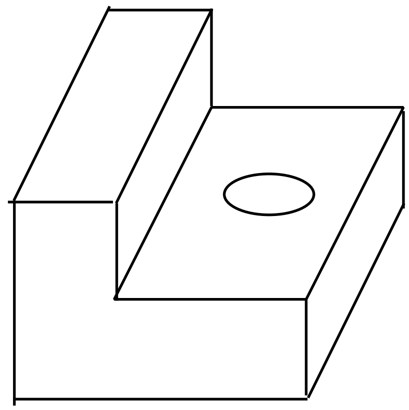
Define path as a curve represented as a B-spline



Constructive Solid Geometry

Intersection, union and difference of specified volumes

Volumetric rather than surface representation



Form by union of a block lying flat and a block lying on edge

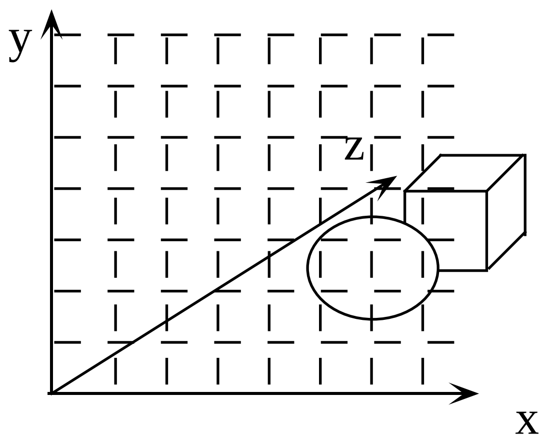
Subtract a cylinder from the union to get the hole

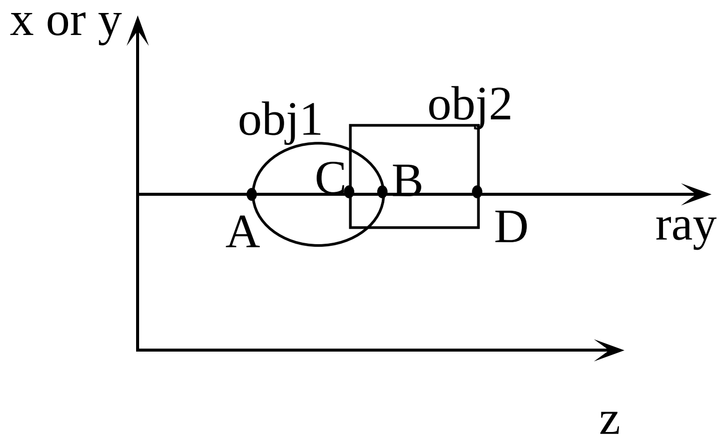
How get a quarter cylinder of height H
from a rectangular block of height H
and a cylinder of height J?

How implement constructive solid geometry in computer graphics system?

If use boundary representation of objects
use Ray-casting

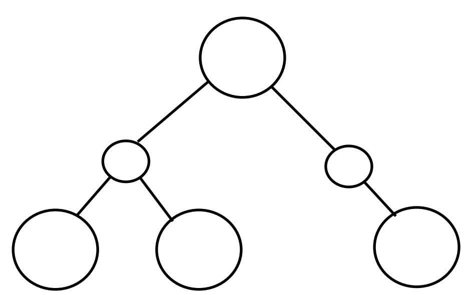
Represent scene in world coordinates
Represent firing plane with x-y plane
Fire rays from each pixel in firing plane
Note intersection of rays with surfaces of objects
Apply operations to intersections to get new boundaries




| Operation | Surface Limits |
| --- | --- |
| Union | A, D |
| Intersection | C, B |
| Difference | |
| obj2-obj1 | B, D |
| obj1-obj2 | A, C |

Easy to represent finished objects as a tree

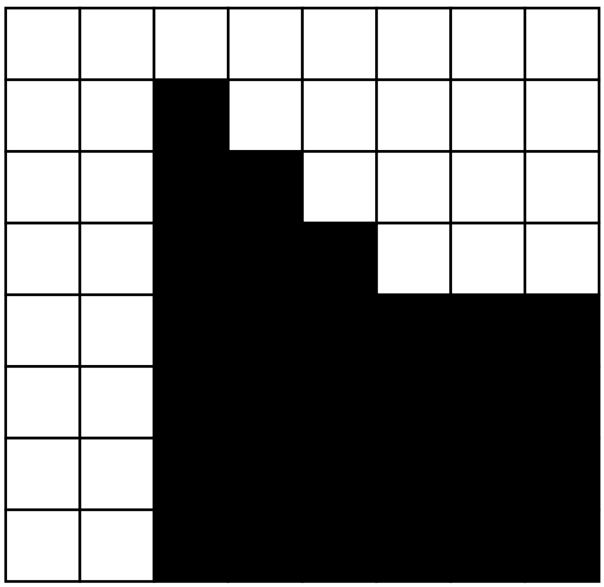


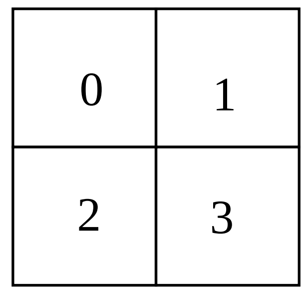
large circles - objects
small circles - operations

# Octrees

Spatial occupancy array

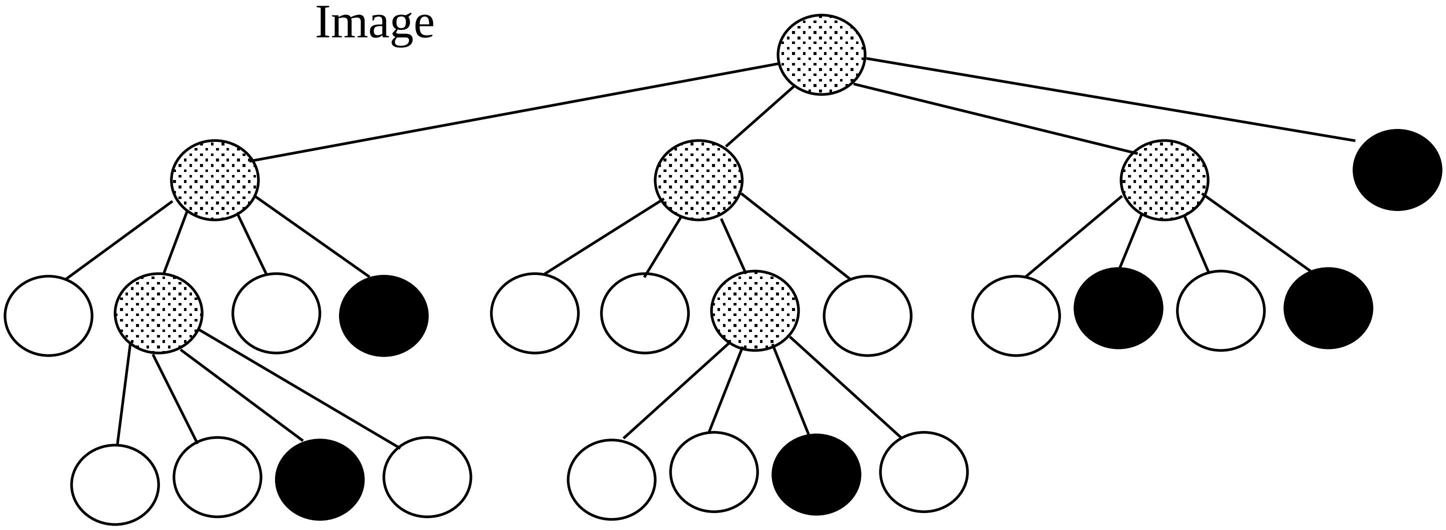
2-D - quadtrees






Quadrant labels

Image



Three types of nodes - white, black, grey

What color are leaf nodes?
What colors can root nodes be?

BSP Trees

- Binary space partitioning
  - Instead of dividing space into 8 regions at each step
    - divide into just two regions using a plane
  - What additional info must be stored?
    - the definition of the partitioning plane
- Can be much more efficient

What happens to the octree representation of an object

if the object is translated?

For a BSP tree?

For image = $2^n \times 2^n$ pixels

What is the minimum height of it's quad-tree?

What is it's maximum height?

Are there always fewer nodes than pixels?

What would be a worst case image?

a checkerboard with square size of one pixel

What number of nodes in a worst case quadtree?

Octrees

Do same thing with voxels versus pixels

Max possible children / node?