Query rewriting

Constraints in clausal form (disjunctions of literals).

Residues

- associated with single literals $p(\bar{x})$ or $\neg p(\bar{x})$ (one of each for every database relation $p$)
- for each literal and each constraint that contains a complementary literal (after renaming), the local residue is obtained by removing the complementary literal and the quantifiers for its associated variables
- for each literal, global residue = conjunction of local residues.

Functional dependencies

$$(\forall x, y, z, y', z')(\neg E(x, y, z) \vee E(x, y', z') \vee y = y')$$

$$((\forall x, y, z, y', z')(\neg E(x, y, z) \vee E(x, y', z') \vee z = z'))$$

Query

$E(x, y, z)$

Local residues

$$(\forall y', z')(\neg E(x, y', z') \vee y = y')$$

$$(\forall y', z')(\neg E(x, y', z') \vee z = z')$$
Constructing the rewritten query

Literal expansion
For every literal in the original query, construct the expanded version as the conjunction of this literal and its global residue.

Iteration
The expansion step is iterated by replacing the literals in the residue by their expanded versions, until no changes occur.

Query expansion
Replace the literals in the query by their final expanded versions.

Functional dependencies

$$\forall x, y, z, y', z' (\neg E(x, y, z) \lor \neg E(x, y', z') \lor y = y')$$

$$\forall x, y, z, y', z' (\neg E(x, y, z) \lor \neg E(x, y', z') \lor z = z')$$

Query
$$E(x, y, z)$$

Rewritten query
$$E(x, y, z) \land (\forall y', z' (\neg E(x, y', z') \lor y = y') \land (\forall y', z' (\neg E(x, y', z') \lor z = z'))$$

Iteration

Integrity constraints

$$\forall x (\neg P(x) \lor R(x))$$

$$\forall x (\neg R(x) \lor S(x))$$

<table>
<thead>
<tr>
<th>Literal</th>
<th>Residue</th>
<th>First expansion</th>
<th>Second (final) expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(x)$</td>
<td>$S(x)$</td>
<td>$R(x) \land S(x)$</td>
<td>$R(x) \land S(x)$</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>$R(x)$</td>
<td>$P(x) \land R(x)$</td>
<td>$P(x) \land R(x) \land S(x)$</td>
</tr>
<tr>
<td>$\neg R(x)$</td>
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<tr>
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</tr>
</tbody>
</table>

Scope of query rewriting

- queries involving conjunctions of literals (relational algebra: $\sigma$, $\land$, $\neg$) and binary universal integrity constraints [ABC99]
- existentially-quantified conjunctions ($\pi$, $\sigma$, $\land$) and single-key dependencies (under certain syntactic restrictions) [FM05]
M. Arenas, L. Bertossi, and J. Chomicki.
Consistent Query Answers in Inconsistent Databases.

A. Fuxman and R. J. Miller.
First-Order Query Rewriting for Inconsistent Databases.
Full version to appear in JCSS.