

Preferences, Queries, Logics

Jan Chomicki
University at Buffalo

DBRank, August 29, 2011

Plan of the talk

1 Preference relations

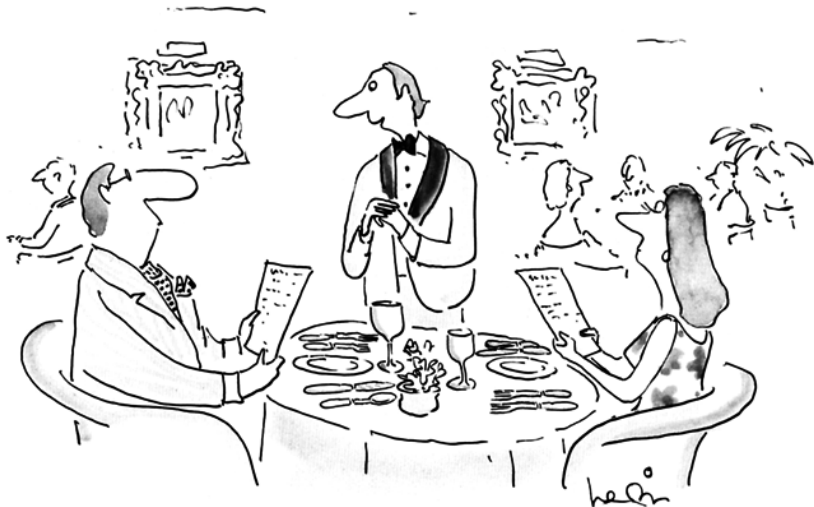
2 Preference queries

3 Advanced topics

Part I

Preference relations

Motivation



“And what is your preference in wine—single or double figures?”

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Preference relations

Universe of objects

- constants: uninterpreted, numbers,...
- individuals (entities)
- tuples
- sets

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Preference relation \succ

- **binary** relation between objects
- $x \succ y \equiv x$ *is_better_than* $y \equiv x$ **dominates** y
- an abstract, uniform way of talking about (relative) desirability, worth, cost, timeliness,..., and their **combinations**
- preference relations used in **preference queries**

Buying a car

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Customer: Wait...it better be a BMW.

Preferences in perspective

Applications of preferences and preference queries

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- 1 decision making
- 2 e-commerce
- 3 digital libraries
- 4 personalization

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Preferences are multi-disciplinary

- economic theory: von Neumann, Arrow, Sen
- philosophy: Aristotle, von Wright
- psychology: Slovic
- artificial intelligence: Boutilier, Brafman
- **databases**: Kießling, Kossmann

Properties of preference relations

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Properties of \succ

- **irreflexivity:** $\forall x. x \not\succeq x$
- **asymmetry:** $\forall x, y. x \succ y \Rightarrow y \not\succeq x$
- **transitivity:** $\forall x, y, z. (x \succ y \wedge y \succ z) \Rightarrow x \succ z$
- **negative transitivity:** $\forall x, y, z. (x \not\succeq y \wedge y \not\succeq z) \Rightarrow x \not\succeq z$
- **connectivity:** $\forall x, y. x \succ y \vee y \succ x \vee x = y$

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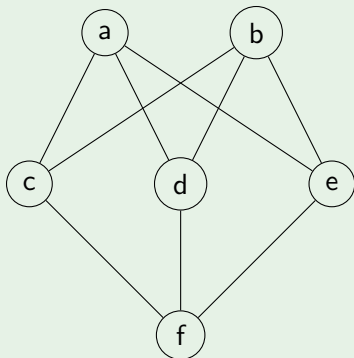
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Orders

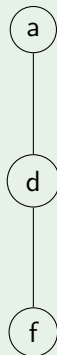
- **strict partial order (SPO):** irreflexive and transitive
- **weak order (WO):** negatively transitive SPO
- **total order:** connected SPO

Weak and total orders

Weak order



Total order



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We assume that preference relations are **SPOs**.

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Canonical example

mazda \succ *kia*, *mazda* \sim^i *vw*, *kia* \sim^i *vw*

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Violation of negative transitivity

mazda $\not>$ *vw*, *vw* $\not>$ *kia*, *mazda* $>$ *kia*

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Explicit preference relations

Finite sets of pairs: $\text{bmw} > \text{mazda}$, $\text{mazda} > \text{kia}$,...

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for relation $\text{Car}(\text{Make}, \text{Year}, \text{Price})$.

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Logic formulas

Logic formulas

The language of logic formulas

- constants
- object (tuple) attributes
- comparison operators: $=, \neq, <, >, \dots$
- arithmetic operators: $+, \cdot, \dots$
- Boolean connectives: \neg, \wedge, \vee
- quantifiers:
 - \forall, \exists
 - usually can be eliminated (**quantifier elimination**)
- no database relations

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Sufficient condition for representability

- $>$ is a weak order
- the domain is **countable** or some **continuity** conditions are satisfied (studied in decision theory)

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Prefer $v \in S_1$ over $v \notin S_1$.

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AROUND(Price, 12K)

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Preference accumulation

- defining preferences over objects in terms of preferences over **simpler objects**
- **dimensionality** is increased (preferences over Cartesian product).

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Boolean composition

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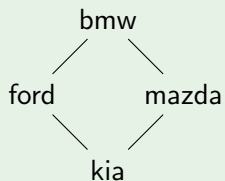
Pareto composition

$$x \succ^{Par} y \equiv (x \succ_1 y \wedge y \not\succeq_2 x) \vee (x \succ_2 y \wedge y \not\succeq_1 x).$$

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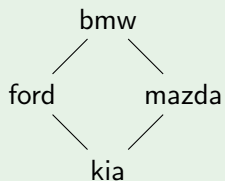
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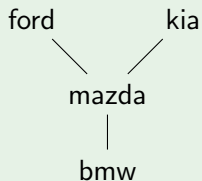


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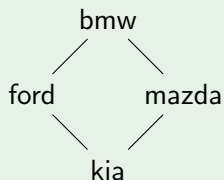


Preference relation \succ_2

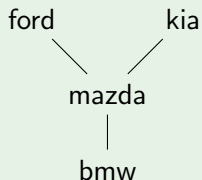


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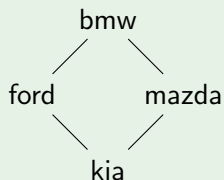


Prioritized composition

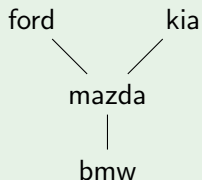


Preference composition

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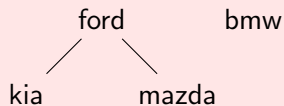
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Prioritized composition



Pareto composition



Combining preferences: accumulation [Kießling, 2002]

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Properties

- closure
- associativity
- commutativity of Pareto accumulation

Skylines

Skyline

Given single-attribute total preference relations $\succ_{A_1}, \dots, \succ_{A_n}$ for a relational schema $R(A_1, \dots, A_n)$, the **skyline** preference relation \succ^{sky} is defined as

$$\succ^{sky} = \succ_{A_1} \otimes \succ_{A_2} \otimes \dots \otimes \succ_{A_n} .$$

Unfolding the definition

$$(x_1, \dots, x_n) \succ^{sky} (y_1, \dots, y_n) \equiv \bigwedge_i x_i \geq_{A_i} y_i \wedge \bigvee_i x_i \succ_{A_i} y_i .$$

Skyline in Euclidean space

Skyline in Euclidean space

Two-dimensional Euclidean space

$$(x_1, x_2) \succ^{sky} (y_1, y_2) \equiv x_1 \geq y_1 \wedge x_2 > y_2 \vee x_1 > y_1 \wedge x_2 \geq y_2$$

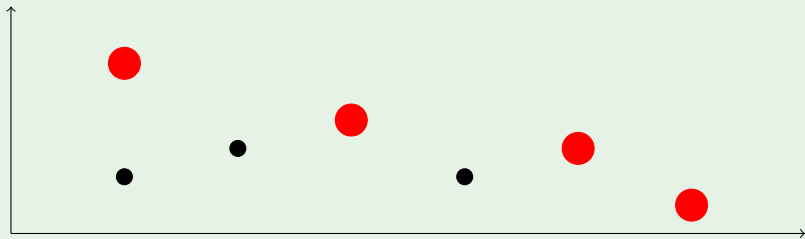
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Maxima

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Skyline is not a weak order

$$(2, 0) \not\prec_{sky} (0, 2), (0, 2) \not\prec_{sky} (1, 0), (2, 0) \succ_{sky} (1, 0)$$

Skyline in SQL

Skyline in SQL

Grouping

Designating attributes **not used** in comparisons (DIFF).

Example

```
SELECT * FROM Car
SKYLINE Price MIN,
         Year MAX,
         Make DIFF
```

Part II

Preference queries

Winnow [Ch., 2002]

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Skyline query

$\omega_{\succ_{sky}}(r)$ computes the set of maximal vectors in r (the **skyline set**).

Example of winnow

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Ranking

Rank tuples by their minimum distance from a winnow tuple:

$$\eta_{>}(r) = \{(t, i) \mid t \in \omega_C^i(r)\}.$$

Generalizations of winnow

Iterating winnow

$$\omega_{>}^1(r) = \omega_{>}(r)$$

$$\omega_{>}^{n+1}(r) = \omega_{>}(r - \bigcup_{1 \leq i \leq n} \omega_{>}^i(r))$$

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k -band

Return the tuples dominated by at most k tuples:

$$\omega_{>}(r) = \{t \in r \mid \#\{t' \in r \mid t' > t\} \leq k\}.$$

Preference SQL

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The language

- basic preference constructors
- Pareto/prioritized accumulation
- new SQL clause PREFERRING
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Winnow in Preference SQL

```
SELECT * FROM Car
PREFERRING HIGHEST(Year)
          CASCADE LOWEST(Price)
```

Algebraic laws [Ch., 2002; Kießling, 2002]

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Commutativity of winnow with selection

If the formula

$$\forall t_1, t_2. [\alpha(t_2) \wedge \gamma(t_1, t_2)] \Rightarrow \alpha(t_1)$$

is valid, then for every r

$$\sigma_\alpha(\omega_\gamma(r)) = \omega_\gamma(\sigma_\alpha(r)).$$

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Under the preference relation

$$(m, y, p) >_{C_1} (m', y', p') \equiv y > y' \wedge p \leq p' \vee y \geq y' \wedge p < p'$$

the selection $\sigma_{Price < 20K}$ commutes with ω_{C_1} but $\sigma_{Price > 20K}$ does not.

Semantic query optimization [Ch., 2004]

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Using information about **integrity constraints** to:

- eliminate redundant occurrences of winnow.
- make more efficient computation of winnow possible.

Eliminating redundancy

Given a set of integrity constraints F , ω_C is **redundant w.r.t.** F iff F implies the formula

$$\forall t_1, t_2. R(t_1) \wedge R(t_2) \Rightarrow t_1 \sim_C t_2.$$

Integrity constraints

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Constraint-generating dependencies (CGD) [Baudinet et al., 1995]

$$\forall t_1 \dots \forall t_n. [R(t_1) \wedge \dots \wedge R(t_n) \wedge \gamma(t_1, \dots, t_n)] \Rightarrow \gamma'(t_1, \dots, t_n).$$

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CGD entailment

Decidable by reduction to the validity of \forall -formulas in the constraint theory (assuming the theory is decidable).

Part III

Advanced topics

Preference modification

Preference modification

Goal

Given a preference relation \succ and additional preference or indifference information \mathcal{I} , construct a new preference relation \succ' whose contents depend on \succ and \mathcal{I} .

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General postulates

- **fulfillment**: the new information \mathcal{I} should be completely incorporated into \succ'
- **minimal change**: \succ' should be as close to \succ as possible
- **closure**:
 - order-theoretic (SPO, WO) properties of \succ should be preserved in \succ'
 - finiteness or finite representability of \succ should also be preserved in \succ'

Preference revision [Ch., 2007]

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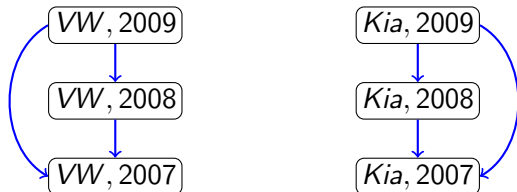
Setting

- new information: **revising** preference relation \succ_0
- composition operator θ : union, prioritized or Pareto composition
- composition eliminates (some) preference conflicts
- additional assumptions: interval orders
- $\succ' = TC(\succ_0 \theta \succ)$ to guarantee SPO

Preference revision [Ch., 2007]

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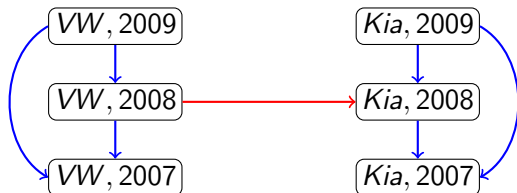
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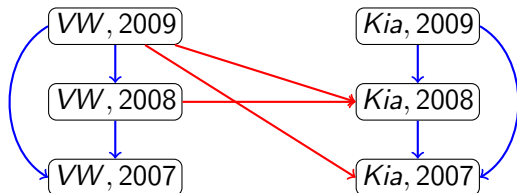
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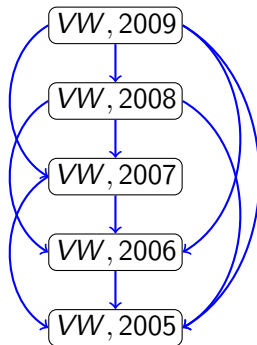
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Preference contraction [Mindolin, Ch., 2011]

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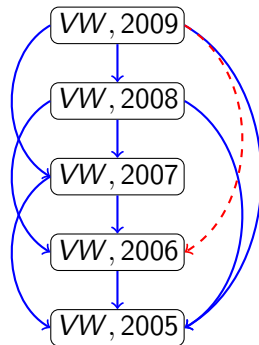
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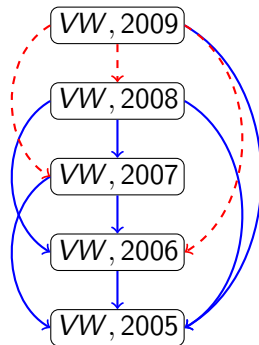
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Substitutability [Balke et al., 2006]

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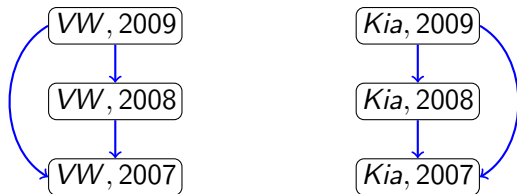
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- additional preferences are added to achieve **object substitutability**

Substitutability [Balke et al., 2006]

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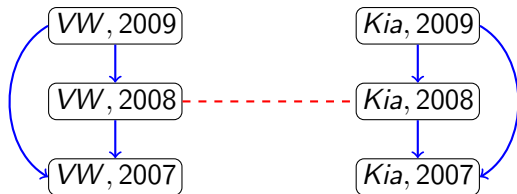
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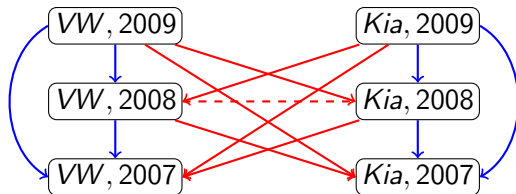
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Preferences over finite sets

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Set preferences

Induced:

$$X \gg Y \equiv \forall x \in X. \exists y \in Y. x > y$$

Aggregate:

$$X \gg Y \equiv \text{sum}_A(X) > \text{sum}_A(Y)$$

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Set preference queries

- find the **best subsets** of a given set
- restrictions on **cardinality**

Preferences over set profiles [Zhang, Ch., 2011]

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Name	Area	Rating
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Preferences

- 2-element subsets
- P_1 : at most one physicist
- P_2 : higher total rating
- P_1 more important than P_2

Set profile (F_1, F_2)

$F_1(S) \equiv \text{SELECT COUNT(Name) FROM S WHERE Area='Physics'}$

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$X \gg_1 Y \equiv F_1(X) \leq 1 \wedge F_1(Y) > 1$

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Prioritized composition

$X \gg Y \equiv X \gg_1 Y \vee (Y \not\gg_1 X \wedge X \gg_2 Y)$

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Incomplete preferences

- tuple scores and **probabilities** [Soliman et al., 2007]
- **uncertain** tuple scores
- **disjunctive** preferences: $a \succ b \vee a \succ c$

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Databases

- preference queries as **decision components**: workflows, event systems
- **personalization** of query results

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Social media

- preference **similarity** and **stability**
- preference **aggregation**

Preference queries vs. Top-K queries

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Preference queries

Binary preference relations

Clear declarative reading

No relational data model extension

Structured data

Manual construction

Preference SQL

Top-K queries

Scoring functions

“Mysterious” formulation

Nondeterminism

Rank-relations [Li et al., 2005]

Structured and unstructured data

Automatic construction

Mainstream DBMS

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- Paolo Ciaccia
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- Xi Zhang

Companion paper

Jan Chomicki, Logical Foundations of Preference Queries, *Data Engineering Bulletin*, 34(2), 2011.

