

CSE4/586 (Spring 2009): Coffee bean problem.

Consider a can C of coffee beans. Each bean is either white or black. We are told the can is initially nonempty. Now consider the program that consists of a single action:

Choose two beans from the can; if they are the same color, toss them out and put in a white bean (there is a sufficient supply); if they are different colors, toss them out and put in a black bean (there is a sufficient supply).

This action is repeated.

1. Let w be the number of white beans in the can and let b be the number of black beans. Write the program corresponding to the previous informal description. (Your program should have only two variables, w and b).
2. Prove that $\{b \geq 0\}$ is an invariant of this program.
3. Find a fix-point for the program.
4. Give a variant function for this program and use it to prove the program terminates.
5. If the program terminates with $b = 1$, what must have been true of the beans in the can at the beginning of the computation? (Hint #1: find an invariant property on w and/or b .) (Hint #2: to gain some intuition, it may be helpful to draw the state space and action transition picture.)