Decision Trees
An Early Classifier

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We cover such problems involving **nominal data** in this chapter—that is, data that are discrete and without any natural notion of similarity or even ordering.

For example (DHS), some teeth are small and fine (as in baleen whales) for straining tiny prey from the sea; others (as in sharks) come in multiple rows; other sea creatures have tusks (as in walruses), yet others lack teeth altogether (as in squid). There is no clear notion of similarity for this information about teeth.
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- Most of the other methods we study will involve real-valued feature vectors with clear metrics.

- We may also consider problems involving data tuples and data strings. And for recognition of these, decision trees and string grammars, respectively.
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- How did you ask the questions?
- What underlying measure led you the questions, if any?
- Most importantly, iterative yes/no questions of this sort require no metric and are well suited for nominal data.
These sequence of questions are a decision tree...

level 0

level 1

level 2

level 3
Decision Trees 101

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- We follow the link corresponding to the appropriate value of the pattern and continue to a new node, at which we check the next property. And so on.
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- Decision trees have a particularly high degree of interpretability.
When to Consider Decision Trees

- Instances are wholly or partly described by attribute-value pairs.
- Target function is discrete valued.
- Disjunctive hypothesis may be required.
- Possibly noisy training data.

Examples
- Equipment or medical diagnosis.
- Credit risk analysis.
- Modeling calendar scheduling preferences.
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- Any decision tree will progressively split the data into subsets.
- If at any point all of the elements of a particular subset are of the same category, then we say this node is pure and we can stop splitting.
- Unfortunately, this rarely happens and we have to decide between whether to stop splitting and accept an imperfect decision or instead to select another property and grow the tree further.
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3. When should a node be declared a leaf?
4. How can we prune a tree once it has become too large?
5. If a leaf node is impure, how should the category be assigned?
6. How should missing data be handled?
Number of Splits

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- Note that any split with a factor greater than 2 can easily be converted into a sequence of binary splits.
- So, DHS focuses on only binary tree learning.
- But, we note that in certain circumstances for learning and inference, the selection of a test at a node or its inference may be computationally expensive and a 3- or 4-way split may be more desirable for computational reasons.
Query Selection and Node Impurity

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- Entropy impurity is the most popular measure:

$$i(N) = - \sum_j P(\omega_j) \log P(\omega_j) .$$  \hspace{1cm} (1)

It will be minimized for a node that has elements of only one class (pure).
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The **misclassification impurity** measures the minimum probability that a training pattern would be misclassified at \( N \):

\[ i(N) = 1 - \max_j P(\omega_j) \]  

(4)
For the two-category case, the impurity functions peak at equal class frequencies.
Query Selection

Key Question: Given a partial tree down to node $N$, what feature $s$ should we choose for the property test $T$?

The obvious heuristic is to choose the feature that yields as big a decrease in the impurity as possible. The impurity gradient is
\[
\Delta_i(N) = i(N) - p_L i(N_L) - (1 - p_L) i(N_R),
\]
where $N_L$ and $N_R$ are the left and right descendants, respectively, $p_L$ is the fraction of data that will go to the left sub-tree when property $T$ is used. The strategy is then to choose the feature that maximizes $\Delta_i(N)$.

If the entropy impurity is used, this corresponds to choosing the feature that yields the highest information gain.
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If the **entropy impurity** is used, this corresponds to choosing the feature that yields the highest **information gain**.
What can we say about this strategy?

- For the binary-case, it yields one-dimensional optimization problem (which may have non-unique optima).

- In the higher branching factor case, it would yield a higher-dimensional optimization problem.

- In multi-class binary tree creation, we would want to use the twoing criterion. The goal is to find the split that best separates groups of the categories. A candidate "supercategory" $C_1$ consists of all patterns in some subset of the categories and $C_2$ has the remainder.

- When searching for the features, we also need to search over possible category groupings.

- This is a local, greedy optimization strategy. Hence, there is no guarantee that we have either the global optimum (in classification accuracy) or the smallest tree.

- In practice, it has been observed that the particular choice of impurity function rarely affects the final classifier and its accuracy.
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