# Laser Based Simultaneous Mutual Localisation for Multiple Mobile Robots 

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#### Abstract

The premise of cooperative localisation is extended by the idea of simultaneous mutual localisation in this paper. Through simultaneous mutual observation robot teams are able to determine their relative poses accurately and robustly. This novel localisation scheme coupled with a custom circular target fitting approach delivers good pose determination with average position and orientation errors of 0.01 m and $1.4^{\circ}$. The experimental results show that our proposed scheme can be used to produce maps with a geometric error of about $2 \%$.

Index Terms-Mutual Localisation, Cooperative Mapping, Laser Scanner, Circle Fitting


## I. Introduction

Mobile robots are expected to operate in a variety of locales, both indoors and outdoors, as well as, structured and unstructured. These environments are usually dynamic and vary enormously. The ability of a mobile robot to locate itself accurately and robustly within these environments is one of the most fundamental problems currently thwarting mobile autonomous operation. Most current localisation methods require a prior map or attempt to build one. Although the problem of locating a robot given a prior map has been solved in many fashions, there is not yet a universally adopted method. Difficulties localising using natural landmarks have resulted in a resurgence in the use of artificial landmarks for localisation [1]. Success using natural landmarks such as corners has been achieved by [2]. There is no doubting the necessity of fast, accurate and robust navigation and mapping approaches that would work in these environments. Better would be the extraction of paradigms that would span all arenas of operation.
One of these paradigms is that some tasks are best tackled by a team of robots rather than just one. Mapping is one of these tasks. For teams of robots to take advantage of the multirobot approach they must be able to locate themselves relative to one another in a fast, accurate and reliable manner. One such way involves robots observing targets with rotational symmetry of order one which are mounted on each robot. This allows determination of the relative pose of the observing robot. Pose errors increase rapidly with range as it heavily depends upon the ability of the observer to calculate the orientation of the observed target. This requires high resolution data of the target which
diminishes linearly with increasing separation. The use of multiple landmarks [3] increases the accuracy of the pose estimate by effectively deploying a large target consisting of two separated landmarks.

In this paper we demonstrate mutual localisation which is more effective than standard cooperative localisation. The pivotal idea for mutual localisation is that rather than one robot acting as an observer and the other as a beacon, both robots act as observers and beacons to one another. Thus two or more robots operating as a team can use these so called simultaneous mutual observation events to precisely determine their relative poses regardless of range. This has a number of immediate significant impacts. Only two robots are required, immunity to environmental distractors and increased relative pose accuracy. The pose error no longer deteriorates with separation and in fact proves to be independent of separation, quite a counter-intuitive result.

The rest of the paper is organised as follows. Section II presents the background to our approach. In Section III the mutual localisation process is detailed. The experimental results are summarised in Section IV and analysed in Section V. Finally the conclusion and further work are discussed in Section VI.

## II. Background

Mutual localisation relies on the ability of each robot to both observe and be observed by other robots. This mechanism is achieved in [4] and [5] amongst others. Omnidirectional vision is employed in [4] which enables relative angles to be available at all times. The range estimates are produced by measuring the apparent colour blob size however they are inaccurate and susceptible to lighting conditions. Laser range finders with two retroreflective cylindrical targets are used in [5] but relative pose is determined by observation of the target's orientation; consequently accuracy deteriorates quite rapidly with range. The ideal set-up would use the laser scanner which benefits from requiring only one sensor and the inherent accuracy of the laser scanning returning both range and bearing as shown in Fig. 1. The two-dimensional nature of the laser scanner and the $180^{\circ}$ field of view are the main disadvantages. The two-dimensional scan means that it


Fig. 1. Pioneer equipped with laser scanner and retroreflective beacon.


Fig. 2. Mechanisms for mutual observation.
is difficult to set up an observable beacon in the plane without blocking some part of the scan.

A number of the mechanisms for mutual observation considered are shown in Fig. 2. The simplest, for theoretical analysis, are set-ups where the position of the beacon on the robot coincides with the origin of the laser scanner. This may be achieved by mounting the laser scanners at different heights, Fig. 2(d), and angling the laser scanners, Fig. 2(a). Alternatively the laser scanner may be encapsulated in a cylinder with a slot cut out allowing the laser rays to escape as shown in Fig. 2(b). Adequate tilting ensures that the laser rays of other robots only perceive the outer cylinder. The main disadvantage of this approach is the variation in tilt forwards as opposed to that at the sides where the rays are horizontal; for fields of view less than $180^{\circ}$ this may not be a problem. Also two targets could be located either side of the scanner, Fig. 3(b), however detection of one of the targets gives rise to an ambiguous pose.

One of the main drawbacks of retroreflective targets is the limited observation angle. In experiments it was observed that retroreflectivity failed for angles of incidence exceeding $50^{\circ}$. This is an important consideration for straight targets however circular targets always present an observable perpendicular surface. This restriction makes set-ups such as (a) and (d) in Fig. 3 unsuitable. Another consideration when selecting appropriate targets is size. The SICK LMS 200 has a maximum resolution with a field of view of $180^{\circ}$ of $0.5^{\circ}$ which produces a ray separation of approximately 1 cm for every metre of range. For example a cylinder of diameter 0.1 m will always be visible up to 10 m .

Having eliminated mutual observation strategies involving


Fig. 3. Overhead view of beacons allowing mutual observation.
tilting or different heights leaves the strategies illustrated in Fig. 3. The straight line target approach suffers from the limited observation angle of retroreflective tape but offers additional information on the orientation of the target and consequently the pose of the observer. Using cylinders means that there will always be a good retroreflective return, however when one cylinder is occluded orientation of the target is unknown. Angling the straight 'wings' back $45^{\circ}$ allows retroreflective returns at most positions but it is more complicated to extract target pose from observation. In this paper the single cylinder approach depicted in Fig. 2(c) and Fig. 3(c) is adopted.

## III. Mutual Localisation

## A. Detecting Circles

Fast circle detection from laser range data is discussed in [3] and the detection process may be dramatically improved by targets with retroreflecting tape. This approach determines the position of the centre of the circle without fully utilising all the circumference data points. The algorithm essentially supplies the points that are on the circumference of the target circle and belongs to a class of detection algorithms. Given the relevant points, finding the centre of that circle is the job of fitting algorithms. Much research has been done on the fitting of circles, [6]. It is perhaps a surprisingly complex feat to achieve due the non-linearity and poor expression of circles in Cartesian coordinates. Because it is easier for humans to visualise in such a coordinate system it is tempting to convert range readings from the laser scanner into Cartesian coordinates. For fitting circular data this turns out to be a mistake.

Leaving the range readings in polar coordinates has a number of advantages. The points have a definite sequence and the function passing through the points is guaranteed to be single valued for all $\theta$; the same cannot be said for points on a circle in Cartesian coordinates. Sequence means that continuous convex surfaces, such as cylinders, in the environment will produce adjacent points in polar coordinates. The equation of a circle at position $(a, b)$ with radius $R$ in Cartesian coordinates is

$$
\begin{equation*}
(x-a)^{2}+(y-b)^{2}=R^{2} . \tag{1}
\end{equation*}
$$

Using standard Cartesian to polar substitutions and application of some trigonometric identities gives the following relationship between $r$ and $\theta$ for the circumference of a circle
with centre $(c, \alpha)$.

$$
\begin{equation*}
r=\frac{R^{2}-c^{2}}{r}+2 c \cos (\alpha-\theta) \tag{2}
\end{equation*}
$$

Solving this for $r$ using the quadratic formula gives

$$
\begin{equation*}
r=c \cos (\theta-\alpha) \pm \sqrt{R^{2}+c^{2} \cos ^{2}(\theta-\alpha)-c^{2}} . \tag{3}
\end{equation*}
$$

Now as the laser scanner detects surfaces nearer to the origin only the solution involving the negative square root need be considered. The Taylor expansion about $\alpha$ illustrates the suitability of using a quadratic fit in polar coordinates to find the centre of the circle.

$$
\begin{align*}
r=c-R- & \frac{\left(c R-c^{2}\right)(\theta-\alpha)^{2}}{2 R} \\
& +\frac{\left(c R^{3}-4 c^{2} R^{2}+3 c^{4}\right)(\theta-\alpha)^{4}}{24 R^{3}}+\cdots \tag{4}
\end{align*}
$$

A good description of polynomial fitting is given in [7]. In general it is possible to algebraically minimise the vertical offsets for the general $k$ th degree polynomial

$$
\begin{equation*}
y=a_{0}+a_{1} x+\ldots+a_{k} x^{k} \tag{5}
\end{equation*}
$$

Doing this on the Cartesian coordinates results in poor results because the vertical offsets are an approximation to the true perpendicular offset. This approximation is better when the gradient is small, an assumption that is valid for circles in laser range data but most definitely invalid when the range data is converted to Cartesian coordinates. In Cartesian coordinates the circles could have large gradients depending on their location within the field of view of the laser scanner. For a second degree polynomial, $k=2$, solving the matrix equation

$$
\begin{align*}
\mathbf{r} & =\mathbf{X a}  \tag{6}\\
{\left[\begin{array}{c}
r_{1} \\
r_{2} \\
\vdots \\
y_{n}
\end{array}\right] } & =\left[\begin{array}{ccc}
1 & \theta_{1} & \theta_{1}^{2} \\
1 & \theta_{2} & \theta_{2}^{2} \\
\vdots & \vdots & \vdots \\
1 & \theta_{n} & \theta_{n}^{2}
\end{array}\right]\left[\begin{array}{l}
a_{0} \\
a_{1} \\
a_{2}
\end{array}\right] \tag{7}
\end{align*}
$$

by inverting $\mathbf{X}$ allows the calculation of the polynomial coefficients as

$$
\begin{equation*}
\mathbf{a}=\left(\mathbf{X}^{T} \mathbf{X}\right)^{-1} \mathbf{X}^{T} \mathbf{r} \tag{8}
\end{equation*}
$$

In our implementation this is solved by the Java matrix (JAMA) package. Once the quadratic coefficients have been found, determination of the minimum is necessary to find the angle along which the centre of the circle lies. Thus the polar coordinates of the circle centre are

$$
\begin{equation*}
(c, \alpha)=\left(a_{0}-\frac{a_{1}^{2}}{4 a_{2}}+R, \frac{-a_{1}}{2 a_{2}}\right) . \tag{9}
\end{equation*}
$$

The range of the circle centre is the range of the quadratic minimum plus the radius of the circle.


Fig. 4. Geometry used to calculate the relative pose.

This only works if three or more points on the circumference of the circle are detected. For the cases of one and two points geometric approaches must be undertaken. In the following experiments if the number of retroreflective returns in a scan is less than three then the result is discarded. Ideally these results should be included because fewer sample points occur at large separations where better orientation accuracy is possible.

It is usually not feasible, physically, to have the origins of the laser scanner and beacons coincident on a robot. If these origins are displaced then calculating the relative pose is complicated. The best strategies, which are most suitable on balance from both a hardware and theoretical perspective, are using thin targets directly in front of the robot or larger targets but at different heights to the laser scanner. The disadvantages of the former are that the laser data from the central angles of the range scan is discarded and thin targets require the robots to be close for simultaneous mutual observations to occur. Displacement of the beacon from the origin of the laser scanner introduces the complications evident in Fig. 4, where the laser scanners are represented as semi-circles with the forward part of the scanner corresponding to the curve of the semicircle and the beacons as solid circles.

In Fig. 4 the robots are at A and C, with C mapping whilst A remains stationary. The pose of A may be constrained to the origin and $x$-axis without loss of generality. The beacons, B and D , are attached to the robots at A and C respectively. The distances AB and AD are both equal to $d$. The separation of the beacons is labelled as $s_{1}$ and $s_{2}$ because $s_{1}$ is the separation as observed by the robot at A and $s_{2}$ is the separation as observed by the robot at $B$, which in practice will not be identical. The robot at A observes the beacon D at range $a$ and angle $\alpha$ whilst that at C observes the beacon B at range $b$ and angle $\beta$. The beacon separation may be calculated from the robot observations by the cosine rule for triangles.

$$
\begin{align*}
& s_{1}=\sqrt{d^{2}+a^{2}-2 a d \cos \alpha}  \tag{10}\\
& s_{2}=\sqrt{d^{2}+b^{2}-2 b d \cos \beta} \tag{11}
\end{align*}
$$

Comparison of the two values $s_{1}$ and $s_{2}$ (which should be approximately equal) allows erroneous localisations to be
detected and rejected. Typical errors stem from distractors in the environment or error sensitive geometric configurations.

To acquire the position of the robot at C it is best to consider the geometry in the complex plane rather than the more immediately apparent geometric methods involving the sine and cosine rules. Using vectors in the complex plane gives two expressions for the position of C found by traversing the two available paths from the origin to C .

$$
\begin{gather*}
C=d+b e^{\imath \theta}  \tag{12}\\
C=a e^{\imath \alpha}+d e^{\imath(\theta-\beta)} \tag{13}
\end{gather*}
$$

Equating (12) and (13) gives the following expression for $\theta$.

$$
\begin{equation*}
e^{\imath \theta}=\frac{a e^{\imath \alpha}-d}{b-d e^{-\imath \beta}} \tag{14}
\end{equation*}
$$

Eliminating $e^{2 \theta}$ from (12) and simplifying gives C in terms of the observed angles, ranges and the parameter $d$.

$$
\begin{equation*}
C=\frac{a b e^{\imath(\alpha+\beta)}-d^{2}}{b e^{\imath \beta}-d} \tag{15}
\end{equation*}
$$

The real and imaginary parts of $C$ give the Cartesian coordinates of the second robot. The orientation is $\theta-\beta+\pi$ with $\theta$ determined from the argument of (14). The complex number calculations are done in Java using the Java Math Expression Parser (JEP) because Java does not natively handle complex numbers.
Once the pose of robot C is acquired its scan data may be used to update the map as described in Section III-B.

## B. Updating the Map

The map is a standard occupancy grid, [8], where each cell contains the probability that the particular space it represents contains an obstacle. The grid is initialised so that all cells contain 0.5 representing complete uncertainty. New sensor data is incorporated via the Bayesian update process described in (16). Occupancy grids are an effective map representation as they are robust, handle uncertainty well and allow fusion of data from different sensors, [9].

$$
\begin{equation*}
P(\text { occupied })=\left(1+\frac{(1-\text { evidence })(1-\text { prior })}{\text { evidence } \times \text { prior }}\right)^{-1} \tag{16}
\end{equation*}
$$

Due to the fidelity and narrow beam width of the laser scanner a straightforward raytrace model was employed. To this effect, when updating the occupancy grid all cells along the path of the laser ray have their probabilities decreased whilst the cell containing the end point of the ray has its probability increased.

One of the severe limitations of the Bayesian update process stems from the independence assumption regarding successive scans. If the robot is stationary then successive scans are clearly not independent and the robot updates the map with ever increasing certainty. Any errors that are a


Fig. 5. Plan of room used in the simulation alongside map produced.
function of the position are permanently recorded and cannot be undone by conflicting observations from other positions.

A solution similar to [10] alleviates this problem. The work in [10] applies to sonar sensors however with suitable modification works well with laser scanners. In practice the laser scanner errors tend to be only dependent on position. Using pose buckets is not as helpful because there are still multiple updates to the map from the same postion albeit at different orientations. Thus a position bucket system is used which only includes sensor data from new positions regardless of orientation. This is a cautious approach based on the premise that it is better to discard possibly useful data than risk irrevocably damaging the occupancy grid. It may take slightly longer to map but it is more reliable.

## C. Dealing with Distractors

Both robots search for regions of high intensity returns indicative of retroreflective objects in the environment. It is assumed that there are relatively few so called distractors as naturally occurring retroreflective materials are rare. The system however is robust to occasionally occurring retroreflective objects because of the mutual nature of the observations. If one or both of the robots detect a spurious retroreflective distractor then the estimated separation of the robots as observed by each robot are likely to differ substantially. Typically it is found that if the difference between $s_{1}$ and $s_{2}$ from Fig. 4 exceeds a threshold of 0.02 m then the occupancy grid should not be updated.

## IV. Experimental Results

## A. Simulation Experiments

Preliminary work was undertaken in the simulation environment Stage, [11]. The physical set-up consisted of the retroreflective cylindrical beacons with radii 0.01 m positioned 0.2 m directly in front of each robot. The quality of a typical map generated is compared with the true map in Fig. 5. Poses where the beacons were separated by less than two metres were discarded. The occupancy grid is a result of 54 updates. The beacons are 0.2 m directly in front of the robots. One of the advantages of simulation is the ready availability


Fig. 6. Mutual localisation for multiple straight trajectories.


Fig. 7. Orientation error for straight trajectories parallel to the $x$-axis.
of the true map allowing an assessment of map fidelity (Fig. 5).

## B. Real Experiments

Localisation performance was assessed by manoeuvring the robot along a series of straight lines. For this experiment the laser scanners were mounted at different heights and retroreflective cylinders 0.04 m in radius were mounted in front $(0.12 \mathrm{~m})$ and above or below the scanner. Deviation of the pose from the straight line quantified both the position error perpendicular to the line and the orientation error. The position results from these experiments are summarised in Fig. 6. The average position error is small at around 0.01 m and the orientation error is approximately $1.4^{\circ}$.

The orientation accuracy is harder to measure because the accuracy of mutual localisation exceeds the accuracy in aligning the robot. There is no dependence of the orientation error on $\theta$ (Fig. 4) and possibly a weak dependence on $b$ (Fig. 4) as shown in Fig. 7.

A square enclosure of internal dimensions 2.42 m by 2.49 m was mapped. Fig. 8 shows the overhead view and corresponding occupancy grid that was produced. The cell size of the occupancy grid was 0.01 m and the dimensions of the


Fig. 8. Overhead view of enclosure 2.49 m by 2.42 m and corresponding occupancy grid generated.
enclosure were found to be 2.42 m by 2.44 m . These results give a geometric error of around $2 \%$.

The experimental set-up involved pioneer robotic platforms with SICK LMS 200 laser scanners depicted in Fig. 1. Cylindrical beacons 1 cm in diameter wrapped in retroreflective tape were mounted 0.07 m directly in front of the laser scanner.

## V. Analysis

The expression for the position of the robot given in (15) depends upon the observed angles and ranges of the mounted beacons as well as the mount point of the beacons on the robot $(d, 0)$ relative to the origin of the laser scan. In order to properly judge the accuracy of this method an error analysis has to be conducted.

$$
\begin{align*}
& \Delta C(a, b, \alpha, \beta, d)= \\
& \frac{\partial C}{\partial a} \Delta a+\frac{\partial C}{\partial b} \Delta b+\frac{\partial C}{\partial \alpha} \Delta \alpha+\frac{\partial C}{\partial \beta} \Delta \beta+\frac{\partial C}{\partial d} \Delta d \tag{17}
\end{align*}
$$

shows the dependence of the error function for $C$ (Fig. 4) upon the errors in $d$ and the observed ranges and angles. Partial differentiation of $C$ with respect to $a, b, \alpha, \beta$ and $d$ illustrates the functional dependence of the error upon these quantities. Finally assume $d$ is small compared to the separation and the following limits ensue.

$$
\begin{gather*}
\lim _{d \rightarrow 0} \frac{\partial C}{\partial a}=\lim _{d \rightarrow 0}\left(\frac{b e^{\imath(\beta+\alpha)}}{b e^{\beta \imath}-d}\right)=e^{\imath \alpha} \\
\lim _{d \rightarrow 0} \frac{\partial C}{\partial b}=\lim _{d \rightarrow 0}\left(\frac{a e^{\imath(\beta+\alpha)}}{b e^{\imath \beta}-d}-\frac{e^{\imath \beta}\left(a b e^{\imath(\beta+\alpha)}-d^{2}\right)}{\left(b e^{\imath \beta}-d\right)^{2}}\right)=0 \tag{19}
\end{gather*}
$$

$$
\begin{equation*}
\lim _{d \rightarrow 0} \frac{\partial C}{\partial \alpha}=\lim _{d \rightarrow 0}\left(\frac{a b \imath e^{\imath(\beta+\alpha)}}{b e^{\imath \beta}-d}\right)=a \imath e^{\imath \alpha} \tag{20}
\end{equation*}
$$

$$
\begin{gather*}
\lim _{d \rightarrow 0} \frac{\partial C}{\partial \beta}= \\
\lim _{d \rightarrow 0}\left(\frac{a b v e^{\imath(\beta+\alpha)}}{b e^{\imath \beta}-d}-\frac{b \imath e^{\imath \beta}\left(a b e^{\imath(\beta+\alpha)}-d^{2}\right)}{\left(b e^{\imath \beta}-d\right)^{2}}\right)=0  \tag{21}\\
\lim _{d \rightarrow 0} \frac{\partial C}{\partial d}=\lim _{d \rightarrow 0}\left(\frac{a b e^{\imath(\beta+\alpha)}-d^{2}}{\left(b e^{\imath \beta}-d\right)^{2}}-\frac{2 d}{b e^{\imath \beta}-d}\right)=\frac{a}{b} e^{\imath(\alpha-\beta)} \tag{22}
\end{gather*}
$$

The error in the position as expressed in (15) not only depends on the partial derivative but also on the errors in the quantities involved.

$$
\begin{equation*}
\Delta a \approx 0.02 m, \Delta b \approx 0.02 m, \Delta \alpha \approx \frac{w}{a}, \Delta \beta \approx \frac{w}{b} \tag{23}
\end{equation*}
$$

where $w$ is the apparent width of the mounted target at that observation angle. In the simple case of a circular target the observed width is always the same. For the far field approximation assuming, in the worst case, only one laser beam is reflected from the target then the error is given by (23). In this case the angular error is proportional to the size of the target over the range. Consequently the total error in $C$ is independent of robot separation. Although the error is independent of robot separation, increased distance between the robots decreases the frequency of simultaneous mutual detection events. Thus reducing the frequency of simultaneous observation pose updates. Asynchronous observations may still be used to track position and orientation separately.

Combining the partial differentials and (23) gives the overall error in the position expressed as

$$
\begin{equation*}
\Delta C=e^{\alpha \imath} \Delta a+a \imath e^{\alpha \imath} \Delta \alpha+\frac{a}{b} e^{(\alpha-\beta)^{2}} \Delta d \tag{24}
\end{equation*}
$$

In the far field approximation with small $d$ when $a$ and $b$ are similar in size the quotient $\frac{a}{b}$ tends to one. The potentially dominating error term involving $\Delta \alpha$ is subdued by the $\frac{1}{a}$ dependence of $\Delta \alpha$. By taking the modulus of each term in (24) and substituting (23) the maximum error is given by

$$
\begin{equation*}
\Delta C_{\max } \approx \Delta a+w+\Delta d \tag{25}
\end{equation*}
$$

This result is important because it demonstrates that error in the far field approximation for large robot separations is not unduly sensitive to the quantities measured for any position or orientation combination for two robots.

For accurate mutual localisation it is vital that the mutual observation events are simultaneous. By simultaneous it is meant that there has been minimal pose change between the times of the observation events. This is especially important for the orientation part of the pose. If mutual observation synchronicity proves difficult a simple solution is to use the pose information whilst moving and only update the map when the robot is stationary or the rate of pose change sufficiently small.

## VI. Conclusion and Future Work

A new approach to cooperative localisation has been explored in this paper. Traditionally cooperative localisation has been the natural extension of the corpus of work on artificial landmarks. Namely that the benefits of artificial landmarks can be enjoyed without the need to deploy them or otherwise alter the environment. Localisation and mapping was accomplished by mounting beacons on robots and using them as artificial landmarks. By leaving the beacon robots stationary an exploratory robot could navigate and map an area. When that area has been mapped the team may move on to another area. By mounting beacons on robots we have not only mobile landmarks but also observing landmarks.

Ensuring that robots may observe team-mates and be observed themselves means that simultaneous mutual localisation events can occur. These events allow superior relative pose determination. Firstly, the mutual localisation is robust to spurious readings because simple checks on the validity of the mutual pose are available; for instance the separation of the robots should be similar as measured by both observing robots. Secondly, the accuracy in the pose does not deteriorate with separation, a very useful property. Increasing separation merely decreases the frequency of mutual localisation events.

Further work will consider scaling this approach to many robot teams and the merging of multiple local maps to produce large global maps. Applying this tool to the problem of mapping in three dimensions will also be explored.

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