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# Fused Multi-sensor Data Using a Kalman Filter Modified With Interval Probability Support

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## Abstract

Multi-sensor fusion problem is mainly composed of three sub-problems: Selection, Fusion and Estimation. Selection is choosing a representative subset of the sensors. Fusion is to take two or more separate sensors data and merge them to form a single entity. Estimation is the process of identifying the features of fused data. This paper addresses the issue of estimating an original system state from fused noisy sensors data by using a Kalman filter modified with interval probability support.

The "measured noise variance" in Kalman filter is varied as the confidence in the fused measurement changes. Confidence is determined by means of Interval Probability(Evidential Reasoning), and has a net effect of increasing the filter gain as the confidence increases. The modified Kalman filter is compared to one with constant noise variance, and shows an increase in estimation performance level.

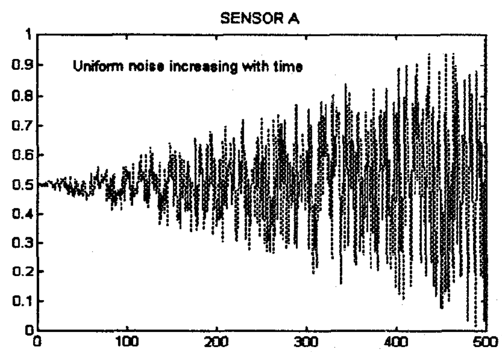
## 1. Introduction

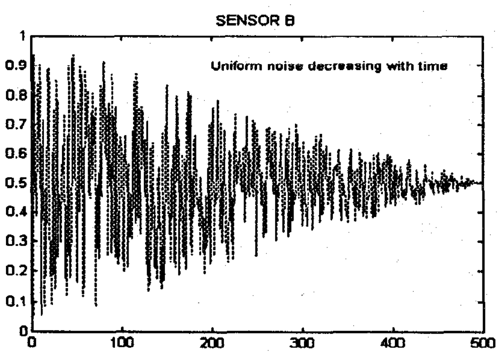
This paper primarily focuses on identifying and estimating the original system state from fused data, selected from noisy sensor measurements. Due to this focus, both Selection and Fusion are kept as simple as possible. The approach presented here can accommodate 'm' sensors reading the state of n-th order system, where m,n=0,1,2,3,......... In order to implement our approach, we illustrate a case of six noisy sensor measurements of state, selected using a band-pass technique. Data is fused by averaging the selected sensors. This weighted average is then used by a modified Kalman filter to estimate the state.

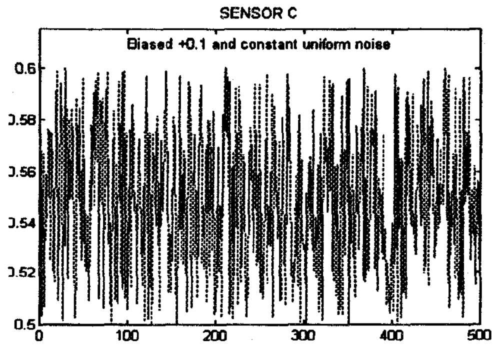
The Kalman filter needs information about the expected noise(Q) in the sensor measurements. This information must be assumed, and should be changed for better performance. Several measurements are taken at any particular time, and the concept of interval probability yields a confidence in measurement. Confidence can be then used to implement a suitable Q. This has an overall effect on the filter gain, which is decreased when confidence is low, and increased when confidence is high.

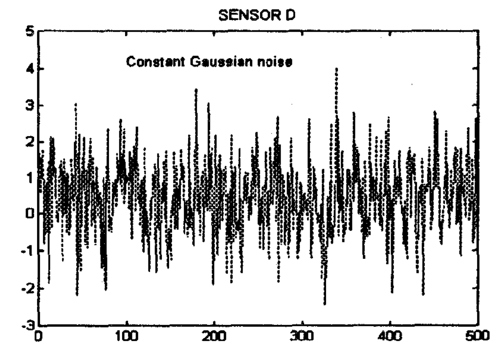
## 2. Sensor Measurements

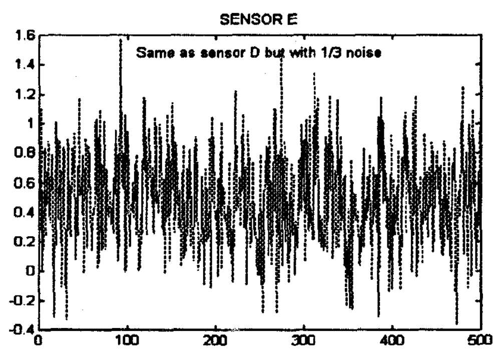
Sensor measurements are generated by using a random(uniform & Gaussian) noise to corrupt the state. Each sensor measurement and the respective noise description are shown below:

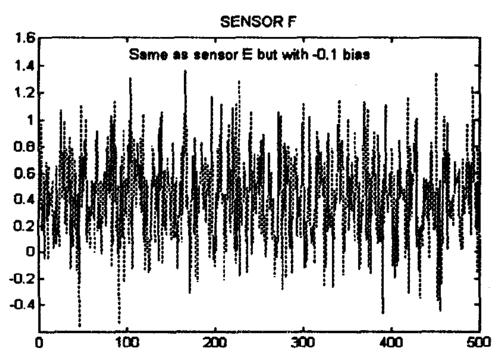




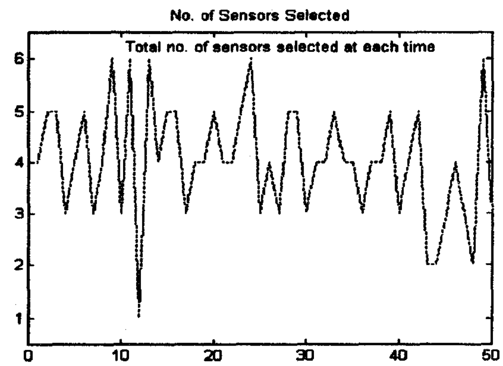



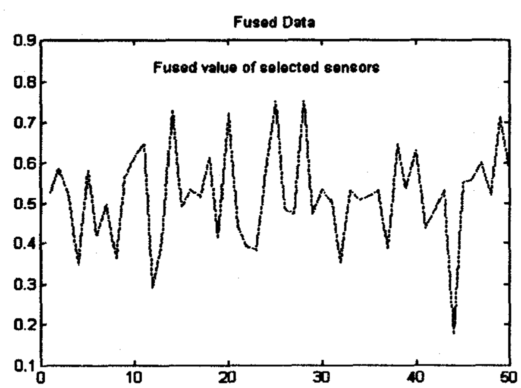

Theoretically, the system is measured every second from 1 to $t_f$ (chosen to be 500). These measurements are placed in a matrix called "$Y_m$".

## 3. Selection and Fusion of Sensors

Selection of sensors is done through a set of rules , which have to be satisfied by a sensor measurement at the time of selection. In this paper, a band-pass filter is used as selector. From the matrix "$Y_m$", a row vector of their weighted averages is formed. This vector is called "$Y_{mav}$". The bandwidth of the band-pass filter is chosen to be threshold $\alpha = \pm 0.3$ deviation from "$Y_{mav}$". The sensor is not selected for fusion if its data lies outside the band. The number of sensors selected for fusion at any particular time is shown below.




After sensors are selected , the next step is to merge their outputs in a single entity. Several methods exist for fusing sensor data depending upon the situation. If two sensor observations are complementary and describe similar but independent situations, then they can be logically added without the concern for conflict. More complex situations arise when multiple sensors are reading the same property value of the same object. In these situations, information must be collectively merged. Several merging methods are available including averaging, and decision between intersecting observations. A graph of the data fused at each particular time is given below:



## 4. Mass Function Generation

Each measurement of a selected sensor is rounded to value between 0 & 1. An upper triangular matrix representing a mass function is formed with a 'ridge' centered at the original measurement of the selected sensor which consists of values higher than the other base values in the matrix. It tapers from the value on the diagonal to the upper right corner. The sum of the elements is normalized to 1, and the ridge-to-base ratio is chosen to be 2-to-1. A typical mass function looks like

$$M=\begin{bmatrix} b & b & b & b & b & b & b & b & b & 2b \\ 0 & b & b & b & b & b & b & b & b & 2b \\ 0 & 0 & b & b & b & b & b & b & 2b & b \\ 0 & 0 & 0 & b & b & b & b & 2b & b & b \\ 0 & 0 & 0 & 0 & b & b & 2b & b & b & b \\ 0 & 0 & 0 & 0 & 0 & 2b & b & b & b & b \\ 0 & 0 & 0 & 0 & 0 & 0 & b & b & b & b \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & b & b \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & b \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b \end{bmatrix}$$

Where 'b' is the base value, calculated so that the sum is '1'. This has the following advantages:

1) Easy to calculate
2) Fast computations
3) Edges go to zero independent of range
4) Compatible with other mass functions
5) Does not favor the corner representing ignorance.

***Combination of mass functions using Dempster's rule:*** Dempster's rule is a method used for blending(fusing) two mass functions into a single mass function. The resulting mass function contains both ridges from the original two, and still sums to '1'.

According to Dempster's rule, two mass functions M1 and M2 can be blended into M3 as follows:

$M3(Q)=1/(1-k)\ \Sigma\ M1(Fi).\ M2(Fj)$, where $Fi \cap Fj=Q$

That is each element in M3 is the product of same elements in M1 and M2, and then normalized by 1/(1-k). where 'k' is defined as below:

$k=\Sigma\ M1(Fi).\ M2(Fj)$, where $Fi \cap Fj=\varnothing$

Each element of M1 is multiplied with every element of M2 except for the corresponding element, and then all the products are added.

***Support Generation:***

The interval probability support of a fused value 'yk' is calculated using the approach described in[2]:

Let k=round(yk)*10 (converts 'yk' to matrix index)

Support is then found from $k-\beta$ to $k+\beta$, where $\beta$=one half the interval size (chosen to be 0.3). e.g. for a fused value of '0.51', after rounding it to the nearest .1 which is 0.5, the support is [0.2 0.8] in the Dempster mass function related to that measurement. Using the above approach, the blended mass function 'M3' (of two mass functions M1 & M2) gives a matrix of support values 'S' as given below:

S=T(M3)T, Where T is an upper triangular matrix consisting of 1's, then spt(i,j)=S(i,j).

## 5. Kalman Estimation

For the sake of comparison, a Kalman filter with associated gain k1=1, is applied with Q=10(noise covariance) such that kQ=constant=10. For both Kalman filters, gain was chosen to be 10(P=KP=10). It was also assumed that there is no system state noise. A vector is generated from the constant Q estimator. The plot of this estimate is shown below.

The second Kalman filter used the interval probability support to update Q, which in turn changed filter gain k2. For a range of K[0 to 1] a desired Q is:

K=P/(P+Q)=KP/(KP+Q)=10/(10+Q).

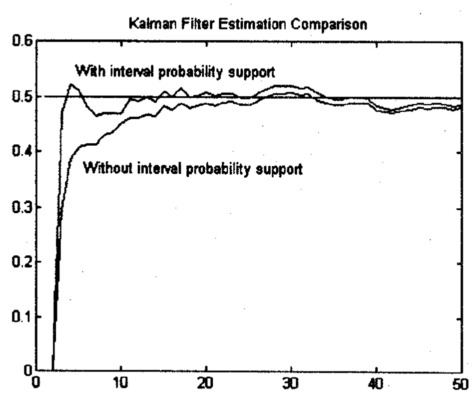
Observing ranges in support averaged 0.08 with 0.15 indicating high confidence and 0.04 indicating low confidence. These values indicate an exponential relationship: Q1= exp(exp(spp)-1)-1

Where spp=(0.21-spt*10)

This relationship worked well except for when error became small at time away from '0'. An alignment of poor measurements(high support) could kick the estimator away from actual state and lower the gain for subsequent iterations, leaving a high error. To correct this, Q is forced to KQ as time increases:

Q2= (Q1+t)/t .KQ.

The Kalman filter then uses this Q2 to estimate the state. Again a vector is generated as the estimate of system state(0.5). State estimates of both versions of Kalman filter are shown below:



## Conclusion

Sensor selection, data fusion followed by Kalman filter, modified with interval probability support performed better most of the time . This indicates that the method of updating Q with interval probability have potential applications. Calculation of the support must be considered when weighing the benefits of interval probability methods in real time. Future research could be directed to explore results with dynamic and possibly nonlinear systems.

A straight forward method of determining confidence is to sum the difference between sensor measurements, at a given time. Interval probability, however, has the ability to generate a variety of shapes of mass functions depending on the knowledge of the sensors.

In this paper, gain varies with the support. Other methods of varying gain could be dependent on plausibility or a combination of plausibility & support. Additional information can also be derived. For example, in target tracking, a confirmed position could be indicated by the support being above a certain threshold confidence. In other system applications, the interval could be decreased until a certain confidence level is reached. This reduces wastage of resources on known areas where target does not possibly reside.

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