

# On the Influence of Self-similarity on Optical Burst Switching Traffic

M. Izal and J. Aracil

Universidad Pública de Navarra. 31006 Pamplona, SPAIN

**Abstract**—In this paper we provide a characterization of OBS traffic (burst size, interarrival time and scaling behavior) when the input traffic is long-range dependent. The analysis shows that the influence of self-similarity on blocking probability is negligible, since the arrival process can be assumed to be Poisson in the timescale of interest for burst blocking. However, the impact for optical buffer dimensioning is significant. On the other hand, the scaling region is shifted to larger timescales while traffic variability at low timescales is increased. These findings serve to accurately dimension number of output ports and optical buffers in OBS routers when the incoming traffic comes from a large population of Internet users.

**Keywords:** OBS, Self-similarity, Fractional Gaussian Noise

## I. INTRODUCTION AND PROBLEM STATEMENT

Optical Burst Switching (OBS) [1] provides "coarse packet switching" service in the optical network, namely a transfer mode which is halfway between circuit switching and pure packet switching. First, a control packet is sent along the path from source to destination in order to reserve resources for the incoming burst. Then, the data burst follows after a short offset time. On one hand, OBS does not incur in the overhead of a (two-way) circuit setup as in a pure circuit switching network. On the other hand, buffer requirements at the intermediate nodes are drastically reduced in comparison to a pure packet switching network, since the switching matrix can be arranged in advance in order to avoid buffering for the incoming burst. Furthermore, OBS offers scope for differentiated quality of service, (MPLS) traffic engineering and path protection and restoration.

Since OBS networks are based on unconfirmed resource reservation, input bursts are subject to blocking in the OBS routers along the path from source to destination. In order to achieve a target blocking probability, the two parameters of interest for router dimensioning are number of output ports (usually wavelengths per fiber) and optical buffers. For such dimensioning purposes, it is usually assumed [2], [3] that the burst arrival process is Poisson and that the burst length distribution is negative exponential, hypergeometric or Pareto. Consequently, the router occupancy can be modeled as a birth-death process and closed expressions for blocking probability can be obtained from Erlangian analysis. However, to the best of our knowledge, there are no papers that actually analyze if the Poisson assumption holds and what is the burst size distribution if the input traffic to the optical cloud does not have independent increments. In this paper, we provide a characterization of OBS traffic assuming, in accordance to widely accepted traffic measurements [4], that the OBS network carries long-range dependent traffic. Our objective is to determine to which extent the self-similarity features of incoming traffic affect OBS traffic

and, ultimately, QoS parameters such as blocking probability.

Fig. 1 shows the scenario under analysis. Since incoming traffic to the OBS cloud comes in packets, burst assembly functionality is required at the edges. We will denote the interworking units in charge by OBS *edge shapers*. An OBS edge shaper maintains a separate queue per Forward Equivalence Class (FEC). One possible alternative for implementation is *Labeled OBS*, an extension of MPLS for OBS networks. We consider traffic characterization at the input of the OBS router (as indicated in Fig. 1), noting that such traffic constitutes the offered load to the optical network.

The traffic characteristics of OBS traffic will be highly dependent on the burst assembly algorithm. Ge, Callegati and Tamil have recently proposed an algorithm for burst assembly [5] that works as follows: Incoming packets to the edge shaper are demultiplexed according to their destination in separate queues. A timer is started with the first packet in the queue. Upon timeout expiration, the burst is assembled and relayed for transmission. As a result, the burst assembly time is kept within the timeout value independent of network load. In doing so, large packetization delays due to burst assembly are avoided, thus circumventing a major drawback of OBS. Furthermore, the fact that bursts are variable length is in accordance with the OBS paradigm [1]. On the other hand, prior to burst transmission, the signalling agent is provided with the burst size and a control packet is released. An offset time later the burst is transmitted to the OBS router. We will assume that the offset time is short and constant, namely the edge shaper provides a single QoS class. Thus, the influence of the offset time in the data burst arrival process is only a time shift.

In this paper, we study the offered traffic to the OBS node

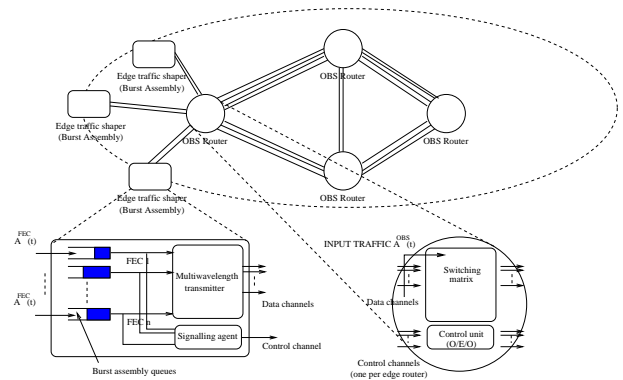


Fig. 1. Network reference model

assuming that edge shapers are loaded with long-range dependent traffic. Our findings show that despite of long range dependence the burst arrival process can be assumed to be Poisson in low timescales. As a consequence, there is nearly no influence of self-similarity on blocking probability, while the influence is significant in optical buffers dimensioning. Finally, we show that OBS traffic inherits the self-similarity of the input traffic but the scaling region is shifted to larger timescales while marginal distribution variability at smaller timescales is increased.

### A. Traffic self-similarity (scaling)

Traffic self-similarity (or scaling) is defined as follows: Let  $\{Z(t), t \in R\}$  be the continuous process of number of bytes arriving in the interval  $[0, t]$  and consider the discrete-time process  $\{X_k = Z(k\delta) - Z((k-1)\delta), k \in N, k \geq 1\}$ , being  $\delta$  a measurement interval. Note that  $X$  denotes the (stationary) discrete process of number of bytes per time interval  $\delta$ . Now, consider the *aggregated* process  $X_i^{(n)} = \frac{1}{n} \sum_{k=n(i-1)+1}^{ni} X_k$ , with  $n > 1, i \geq 1$  and let  $\rho^{(n)}(j)$  with  $j > 1$  be the autocorrelation function of  $X_k^{(n)}$ . The process  $X_k$  is *asymptotically second-order self-similar* if  $\lim_{n \rightarrow \infty} \rho^{(n)}(j) = \frac{1}{2}((j+1)^{2H} - 2j^{2H} + (j-1)^{2H})$  where  $H$  is the Hurst (or self-similarity) parameter. For  $1/2 < H < 1$  the autocorrelation function decays slowly, thus being not summable, and we call  $X_k$  *long-range dependent*.

Note that  $n$  defines a traffic timescale. On the other hand, self-similarity is an asymptotic property, namely, it only happens when  $n \rightarrow \infty$ . In practice, there is a cutoff timescale ( $\delta$ ) beyond which the traffic behaves as a stationary Gaussian self-similar process with constant  $H$  parameter, while the short timescales show complex, multifractal behavior. For timescales beyond such cutoff value the number of bytes per interval are well modeled by a Fractional Gaussian Noise (FGN)<sup>1</sup>. We note that a single arrival process cannot provide a traffic characterization at all timescales. Intuitively, the number of packets per interval can be arbitrarily small if we select a timescale small enough. Hence, for a very short timescale the marginal distribution of the arrival process is not Gaussian but discrete. As we increase the timescale, by the Central Limit Theorem, the statistical multiplexing of packets coming from a larger number of sources results in a Gaussian process. As the network bandwidth increases more packets from different sources can be accommodated in smaller timescales. Thus, the cutoff timescale is expected to decrease with the increasing bandwidth brought by optical technology.

On the other hand, for packet-switched networks the traffic dynamics at low timescale are relevant, specially at low or intermediate load [7]. However, traffic is aggregated in bursts in an OBS network. Therefore, the traffic dynamics at short timescales are not relevant since the burst assembly process can be viewed as a low-pass filter for packet arrival variability at

short timescales. We note that the minimum timescale that we consider in this paper is the burst assembly timeout. Consequently, we may safely assume that, for such timescales, the incoming traffic to the optical cloud behaves as a FGN [6].

The rest of the paper is organized as follows: in section I-B we define the methodology, section II presents the analytical models, followed by the results and discussion in section III. Finally, we present the conclusions that can be drawn from this paper.

### B. Methodology

We will first assume that the traffic carried by each FEC is a FGN and that FGNs are independent one another. The OBS network supports several FECs from different edge OBS shapers which are assumed to have the same mean  $\mu$ , variance  $\sigma^2$  and Hurst parameter  $H$ , without loss of generality. The FGN parameters are set to those inferred from the *Bellcore traces* (coefficient of variation  $c_v^2 = \sigma^2/\mu^2 = 0.1$ , Hurst parameter  $H = 0.78$ ), which have also been used in other studies [6], [8], [9]. The mean  $\mu$  is a position parameter for the Gaussian distribution and is set to different values in order to simulate different load conditions. We will denote by  $\{A^{FEC}(t), t > 0\}$  the bytes arrival process to a FEC burst assembly queue in the interval  $(0, t]$  (see Fig. 1). Finally, recall that the burst assembly algorithm is timer-based: upon arrival of the first packet in a FEC a timer is started and the burst is ready for transmission once the timeout expires, as proposed in [5]. The timeout value is a parameter in our analysis, which is set to 2 and 4 ms. Anyway, an arbitrary timeout value can be selected since our formulae provide explicit expressions for the traffic model as a function of the timeout value, among others.

## II. ANALYSIS

First, the burst size is evaluated, followed by the burst inter-arrival time. Then, the OBS traffic self-similarity features are examined. The following preliminary lemma is used throughout the section.

**Lemma 1.** Let  $\{Z(t), t \geq 0\}$  be a standard Fractional Brownian Motion with Hurst parameter  $H$  ( $1/2 \leq H < 1$ ). Let  $\{A(t) = \mu t + \sigma Z(t), t \geq 0\}$  be the Gaussian self-similar process that represents the number of bytes arriving in interval  $(0, t]$ . Then,  $A(t + \Delta t) - A(t)$  with  $\Delta t > 0$  is a Gaussian random variable with mean  $\Delta t \mu$  and variance  $\Delta t^{2H} \sigma^2$  [6].

### A. Burst size distribution

Let the timeout value be equal to  $T_{out}$  seconds. Then, the number of bytes per burst from FEC  $i$  is equal in distribution to  $A_i^{FEC}(T_{out}) - A_i^{FEC}(0)$  due to the stationarity of  $\{A_i^{FEC}(t), t \geq 0\}$ ,  $i \in I$ , being  $I$  the set of FECs supported by the OBS router. By direct application of lemma 1 the distribution is Gaussian with mean  $T_{out} \mu$  and variance  $T_{out}^{2H} \sigma^2$ .

<sup>1</sup>An FGN is defined as the increments of a Fractional Brownian Motion [6].

### B. Burst interarrival time distribution

First, we show that the probability of interarrival times larger than  $T_{out}$  is very small. Let  $m$  be the number of FECs and  $T$  the random variable that defines the burst interarrival time at the OBS router, with  $T \geq 0$ . Now consider the events  $M_\tau = \{\text{"No burst arrivals to the OBS node in time } (t, t + \tau)\}$  and  $N_\tau^i = \{\text{"No packet arrivals from FEC } i \text{ in time } (t, t + \tau - T_{out})\}$  for  $1 \leq i \leq m$  and  $\tau > T_{out}$ . We note that  $M_\tau \subset \bigcap_{i=1}^m N_\tau^i$  and, thus,  $P(M_\tau) = P(T > \tau) < \prod_{i=1}^m P(N_\tau^i)$ . Now  $P(N_\tau^i)$  can be obtained by assuming that FEC  $i$  does not receive traffic in a time interval  $\tau$  if the corresponding traffic  $\{A_i^{FEC}(t), t \geq 0\}$  fulfills  $A_i^{FEC}(\tau) - A_i^{FEC}(0) \leq 0$ . By lemma 1,  $A_i^{FEC}(\tau) - A_i^{FEC}(0)$  is a Gaussian random variable with mean  $\tau\mu$  and variance  $\tau^{2H}\sigma^2$ , so that

$$P(N_\tau^i) \approx \exp\left\{-\frac{\mu^2}{2(\tau - T_{out})^{2H-2}\sigma^2}\right\} \quad (1)$$

using the well-known approximation for the tail of a Gaussian distribution. Therefore

$$P(T > \tau) = P(M_\tau) < \exp\left\{-\frac{\mu^2 m}{2(\tau - T_{out})^{2H-2}\sigma^2}\right\} \quad (2)$$

and since  $1/2 < H < 1$  then  $P(T > \tau)$  decays exponentially with  $m(\tau - T_{out})^{2-2H}$ . We note that  $P(T > \tau)$  decays as  $m$  increases, being  $m$  the number of FECs. Consistent with the expected bandwidth increase and processing power of OBS routers, it is reasonable to assume that  $m$  will be large and the last probability negligible. Nevertheless, we perform simulations with  $m$  small (in the tens) in the next section and check that the probability of interarrival times larger than  $T_{out}$  is negligible.

Consequently, on the other hand, the number of burst arrivals at timescales  $\tau < T_{out}$  can be approximated as a Binomial random variable  $B(m, p)$  with  $p = \tau/T_{out}$ . Let  $Y(\tau), \tau \geq 0$  represent the number of burst arrivals in  $\tau$ , then the burst interarrival time distribution can be obtained as

$$P(T > \tau) = P(Y(\tau) = 0) \approx (1 - \tau/T_{out})^m \quad 0 \leq \tau \leq T_{out} \quad (3)$$

At very short timescales we note that  $\tau/T_{out} \rightarrow 0$  and, if  $m$  is large enough so that  $m\tau/T_{out} \rightarrow \lambda$ , being  $\lambda$  a positive constant, the Binomial distribution can be approximated by Poisson with parameter  $\lambda$  and the interarrival time is negative exponential. For larger timescales, as we have seen before,  $T$  converges in distribution (with  $m$ ) to a random variable with support  $(0, T_{out})$ , while the support of the exponential random variable is  $(0, \infty)$ . Therefore, the Poisson assumption tends to overestimate large interarrival times. However, (3) is an approximation of order  $m$  of  $\exp[-m\tau/T_{out}]$ . Therefore, for large  $m$  we may assume that interarrival times are exponentially distributed in the whole interval  $(0, T_{out})$ , the better the approximation the lower the interarrival time.

### C. Self-similarity and marginal distribution variability

Even though OBS traffic can be considered as Poisson (Poisson arriving Gaussian size bursts) in low timescales, traffic remains self-similar in timescales beyond  $T_{out}$ . In order to evaluate self-similarity we choose the popular variance-aggregation plot for the bytes arrival process. Choosing the bytes arrival process has a twofold advantage: First, it allows to check to which extent the OBS traffic  $A^{OBS}(t)$  inherits the self-similarity features of the input traffic  $A^{FEC}(t)$ , since they both come in the same units. Secondly, it allows to characterize the OBS router offered load: Let  $C$  be the aggregate capacity (bps) of an output fiber in the OBS router<sup>2</sup>, then  $A^{OBS}(t)/(t\eta C)$  is the offered traffic per fiber (Erlangs) in the interval  $(0, t]$ , being  $\eta$  the number of fibers and assuming uniform destinations. The offered traffic determines the router blocking probability. For a Poisson arrival process the offered traffic has independent increments. However, the offered traffic to an OBS router does not have independent increments, as we will show in this section, and that has an impact in blocking probability. On the other hand, the use of a variance-aggregation plot is convenient since not only an estimation of the  $H$  parameter is provided, but also the timescales for which the process shows scaling behavior can be observed<sup>3</sup>. The  $H$  estimation is obtained with the slope of the least-square regression line of the variance-aggregation plot, which decays as  $2H - 2$  for a long-range dependent process. The estimate proves reliable for an stationary, homogeneous Gaussian process [10, Chapter 4].

Let us consider timescales shorter than  $T_{out}$ . First, we show that OBS traffic is not long-range dependent for such timescales. Then we provide a tight lower bound for variance versus aggregation level. Since the number of burst arrivals in intervals of duration  $\tau < T_{out}$ , which we denote by  $Y(\tau)$ , is approximately given by a Poisson random variable with parameter  $\lambda = m\tau/T_{out}$ , the moments are equal to  $E[Y(\tau)] = \tau\lambda$  and  $E[Y(\tau)^2] = \tau\lambda + (\tau\lambda)^2 = \tau\lambda(1 + \tau\lambda)$ .

On the other hand, the number of bytes per burst has a normal distribution with mean  $T_{out}\mu$  and variance  $T_{out}^{2H}\sigma^2$ . According to our notation, if  $A^{OBS}(t)$  is the number of bytes in  $(0, t]$  then  $A^{OBS}(\tau) = \sum_{i=1}^{Y(\tau)} Z_i$ , being  $Z_i, i = 1, \dots, Y(\tau)$  i.i.d random variables  $N(T_{out}\mu, T_{out}^H\sigma)$ . Then,

$$\begin{aligned} E[A^{OBS}(\tau)] &= E[E[A^{OBS}(\tau)|Y(\tau)]] = E[Z]E[Y(\tau)] \\ E[A^{OBS}(\tau)^2] &= E[Y(\tau)]Var[Z] + E[Y(\tau)^2]E^2[Z] \quad (4) \end{aligned}$$

with  $0 < \tau < T_{out}$  and zero otherwise. Thus,

$$Var[A^{OBS}(\tau)] = \tau\lambda(T_{out}^2\mu^2 + T_{out}^{2H}\sigma^2) \quad (5)$$

and the variance-aggregation plot is obtained by (log-log) plotting  $Var[A^{OBS}(\tau)/\tau]$  versus the timescale  $\tau$  with  $0 < \tau < T_{out}$ . Since

<sup>2</sup>(Number of wavelengths per fiber)x(Wavelength bandwidth (bps))

<sup>3</sup>We have also used wavelet-based estimators and obtained the same results

$$\text{Var}[A^{OBS}(\tau)/\tau] = \frac{\lambda}{\tau} (T_{out}^2 \mu^2 + T_{out}^{2H} \sigma^2) \quad (6)$$

it turns out that  $\text{Log}(\text{Var}[A^{OBS}(\tau)/\tau]) = \text{Log}(K_1) - \text{Log}(\tau)$ , being  $K_1$  a constant, namely the variance of the aggregated process decays linearly (slope -1) with the aggregation level in log-log scales and, consequently, the process does not show long-range dependence for timescales shorter than  $T_{out}$ .

While the Poisson approximation is adequate for timescales shorter than  $T_{out}$  it becomes less accurate as we approach to timescales in the vicinity of  $T_{out}$ , since the number of bursts is  $m$  with probability close to one, following the discussion in section II-B. We now provide a tight lower bound for  $\text{Var}[A^{OBS}(\tau)/\tau]$  in the region  $0 \leq \tau \leq T_{out}$ . To do so, we consider the constant  $\xi \approx dt$  such that the probability of more than one burst arriving in a time interval  $\xi$  is very small. Now, we "slot" the time axis in  $n_{tout}$  slots per timeout interval, being  $n_{tout} = T_{out}/\xi$ . If  $\xi$  is small we can obtain the number of burst arrivals at discrete timescales  $(0, j\xi), j < n_{tout}$  as a hypergeometric random variable  $Y(j)$  with distribution:

$$P(Y(j) = y) = \frac{\binom{j}{y} \binom{n_{tout} - j}{m - y}}{\binom{n_{tout}}{m}} \quad (7)$$

with  $\max(0, m - n_{tout} + j) \leq y \leq \min(j, m)$ . We proceed by substitution of the moments of  $Y(j)$  in (4), as we did before with the Poisson moments. Consider the "slotted" process  $A^{OBS'}(j) = A^{OBS}(j\xi)$  and obtain  $\text{Var}[A^{OBS'}(j)/j]$  versus the timescale  $j$  with  $1 < j < n_{tout}$ , namely

$$\text{Var}(A^{OBS'}(j)/j) = \frac{m}{jn_{tout}} \left\{ \sigma^2 n_{tout}^{2H} + n_{tout}^2 \mu^2 \left( 1 - \frac{jm}{n_{tout}} + \frac{(j-1)(m-1)}{n_{tout}-1} \right) \right\}. \quad (8)$$

Now, for timescales larger than  $T_{out}$  consider the total input traffic before the edge shaper  $A^{FEC*}(t) = \sum_{i=1}^m A_i^{FEC}(t)$ . It can be shown (see the appendix for a proof) that

$$\lim_{t \rightarrow \infty} P \left( \left| \frac{A^{OBS}(t)}{t} - \frac{A^{FEC*}(t)}{t} \right| < \epsilon \right) = 1 \quad (9)$$

for all  $\epsilon > 0$  and, therefore, the variance-aggregation plot of OBS traffic converges in probability to that of the input process as  $t \rightarrow \infty$ . Thus, as the timescale  $t$  increases the variance decays with the power law  $Kt^{2H-2}$  for both processes, being  $K$  a constant and showing the OBS traffic has long-range dependence for timescales beyond  $T_{out}$ .

### III. RESULTS AND DISCUSSION

Following the same structure as in the previous section we first discuss results for burst size, followed by burst interarrival time and, finally, scaling behavior and traffic variability. Overall, theoretical and simulation results show very good agreement.

#### A. Burst size distribution

Fig. 2 shows the probability density function for burst size with timeout values 2 ms (left) and 4 ms (right), *obtained from simulation*. We first note that the burst size distribution is clearly Gaussian with mean and variance as predicted by the analysis and that the influence of self-similarity ( $H$  parameter) is significant, specially as the timeout values grows. This is due to the fact that, as the  $H$  parameter grows, the variance of the traffic sample mean (aggregated traffic) decays *more slowly* than in the independent case ( $H = 0.5$ ). Thus, *the larger the  $H$  parameter the larger the burst size variance*. Since the burst size distribution determines the optical buffering requirements we conclude that self-similarity has a strong impact in the OBS router storage requirements.

#### B. Burst interarrival time

Fig. 3 shows the (log-linear) interarrival time survival function (complementary of the cumulative density function) for different timeout values, following (3) ("Exact" in the figure) and following the exponential approximation  $\exp[-m\tau/T_{out}]$  ("Poisson" in the figure). The value of  $m$  is fixed and the x-axis is  $\tau/T_{out}, 0 < \tau < T_{out}$ , being  $\tau$  the interarrival time. We note that the exponential distribution tends to overestimate large burst interarrival values, which have very small probability, even for small values of  $m$ . In fact, the number of FECs ( $m$ ) is equal to 10 in the simulations and, thus, the simulation results show that the probability of large interarrival times (see (2) and discussion therein) is negligible, even for small number of FECs. We will return to this issue when we compare blocking probability figures for exponential interarrival times versus real interarrival times.

#### C. Self-similarity and marginal distribution variability

Fig. 4 shows analytical and simulation results for the variance-aggregation plot. The Poisson approximation (6) is accurate for small timescales while the Hypergeometric approximation (8) provides a tight lower bound, specially for timescales in the vicinity of the timeout value. For timescales immediately following the timeout value we observe the effect of burst assembly (sidelobes), which tends to smooth out as the timescale increases (9). Overall, the process does not show long-range dependence in timescales smaller than the timeout value and *scales as the original traffic* for larger timescales. Therefore, we observe a *shift* in the scaling region when compared to the original FEC traffic and an *increase* in variability in small timescales, due to the increase in variance of the OBS traffic. On the other hand, the convergence in probability of the sample means of both original and OBS traffic as the timescale increases, predicted by (9) is apparent in the figure. Consequently, as the timescale increases, the difference between both original and OBS traffic, concerning scaling behavior, becomes negligible. The results can be explained as follows: while traffic is not changed significantly for large timescales, in low

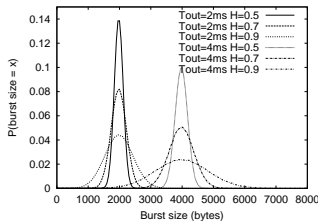


Fig. 2 Burst size distribution

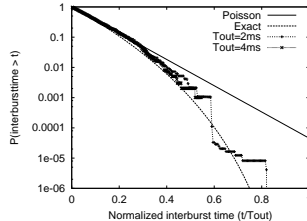


Fig. 3 Burst interarrival times

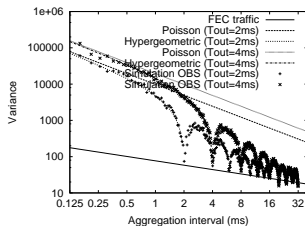


Fig. 4 Variance-aggregation plot

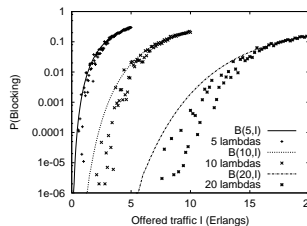


Fig. 5 Blocking probability

timescales the traffic process is “whitened” due to burst sequencing and shuffling at the input of the optical transmission queue. Opposite to this beneficial effect an increase in marginal distribution variability is observed.

#### D. Impact for performance evaluation

In order to provide an example of the impact of traffic modeling in OBS performance evaluation we consider an OBS router with a bufferless symmetric switch engine with  $\eta$  fibers and  $m_\eta$  wavelengths per fiber and full wavelength conversion capability. If the input traffic is Poisson, with uniform destinations and with an RFD-based (reserve-a-fixed-duration) signaling scheme the blocking probability per fiber is given by the Erlang-B formula  $B(m_\eta, I)$  [2] where  $I$  is the offered traffic per fiber, independent of the burst size distribution. We perform simulations with the same mean service time for a burst, same average arrival rate and burst sizes modeled according to section II-A. The results for blocking probability (5, 10 and 20 wavelengths) are presented in Fig. 5.

We observe that the Poisson approximation provides an upper bound for blocking probability, which is tighter as load increases, but is not exact (simulation points fall below Erlang-B curves). This is due to the fact that i) burst interarrival times are not exactly exponential (section II-B) and ii) the offered traffic does not show independent increments for timescales larger than the timeout value (section II-C). However, the relevance of the arrival traffic statistics depends on the timescale of the system under study [11]. For a high-speed network with small or nearly no buffers the short timescales matter and the arrival process can be safely assumed to be Poisson arriving Gaussian bursts for practical engineering purposes.

## IV. CONCLUSIONS

Our findings show that the influence of self-similarity is only relevant for optical buffer dimensioning and negligible for blocking probability, despite of the offered traffic showing long-range dependence in large timescales. These results confirm the use of Poisson arrivals, that is taken as a hypothesis in other papers, and provide explicit expressions for the exact distribution of burst size, interarrival times and scaling behavior (variance versus aggregation level).

## APPENDIX

**Proof of (9):** Let  $\{r_i(t), t > 0\}$  be the backlog of FEC  $i$ ,  $1 < i < m$ , namely the total number of bytes in queue awaiting to be released in a burst. Then,  $\frac{A^{OBS}(t)}{t} = \frac{\sum_{j=1}^m A_j^{FEC}(t) - r_i(t)}{t}$  and, if  $\delta_i^*(t)$  is the time of departure of the last burst at time  $t$  from FEC  $i$  then  $r_i(t)$  is a normal random variable with mean  $\Delta t \mu$  and variance  $\Delta t^{2H} \sigma^2$ , being  $\Delta t = t - \delta_i^*(t)$ . We note that the support of  $\Delta t$  is  $[T_{out}, \infty)$  and  $P(\Delta t > x)$ , with  $x > T_{out}$  is equal to the probability of no packet arrivals in a time interval of duration  $x - T_{out}$ . Following (1), this probability decays exponentially with  $x - T_{out}$  and, therefore  $r_i(t)$  is a normal random variable with bounded mean and variance. As a result,  $r_i(t)/t$  converges in probability to 0 as  $t \rightarrow \infty$  and  $\frac{A^{OBS}(t)}{t}$  converges in probability to  $\frac{\sum_{j=1}^m A_j^{FEC}(t)}{t} = \frac{A^{FEC^*}(t)}{t}$ , and that proves (9).

## REFERENCES

- [1] C. Qiao and M. Yoo, “Optical burst switching (OBS) - A new paradigm for an optical Internet,” *Journal of High-Speed Networks*, vol. 8, no. 1, 1999.
- [2] K. Dolzer, C. Gauger, J. Spath, and S. Bodamer, “Evaluation of reservation mechanisms for optical burst switching,” *International Journal of Electronics and Communications (AE)*, vol. 55, no. 1, 2001.
- [3] M. Yoo, C. Qiao, and S. Dixit, “Qos performance of optical burst switching in ip over wdm networks,” *IEEE Journal of Selected Areas in Communications*, vol. 18, no. 10, pp. 2062–2071, October 2000.
- [4] V. Paxson and S. Floyd, “Wide Area Traffic: The Failure of Poisson Modeling,” *IEEE/ACM Transactions on Networking*, vol. 3, no. 3, pp. 226–244, June 1995.
- [5] A. Ge, F. Callegati, and L. Tamil, “On optical burst switching and self-similar traffic,” *IEEE Communications Letters*, vol. 4, no. 3, March 2000.
- [6] I. Norros, “On the Use of Fractional Brownian Motion in the Theory of Connectionless Networks,” *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 6, pp. 953–962, August 1995.
- [7] A. Neidhart, A. Erramilli, O. Narayan and I. Sanjeev, “Performance impacts of multi-scaling in wide area TCP traffic,” in *IEEE INFOCOM 00*, Tel Aviv, Israel, 2000.
- [8] A. Sang and S. Li, “A predictability analysis of network traffic,” in *Proceedings of Infocom 2000*, 2000.
- [9] S. Ostring and H. Sirisena, “The influence of long-range dependence on traffic prediction,” in *Proceedings of ICC 2001*, 2001.
- [10] J. Beran, *Statistics for long memory processes*, Chapman & Hall, 1994.
- [11] M. Grossglauser and J. Bolot, “On the relevance of long-range dependence in network traffic,” *IEEE/ACM Transactions on Networking*, vol. 7, no. 5, pp. 629–640, 1999.