CSE 451, Fall 1999

CHECKLIST FOR VERIFYING A LOOP GIVEN Q,P,t,R
(Gries, p. 145 (11.9))

Given a loop program, i.e.:

$\{Q\}$
initialization
$\{$invariant $P\}$
$\{$bound function $t\}$
do $\{P\}\ B_1 \rightarrow S_1\ \{P\}$
$[]$ $\quad \ldots$
$[]$ $\quad \{P\}\ B_n \rightarrow S_n\ \{P\}$
od
$\{P\ \wedge\ \neg(B_1 \vee \ldots \vee B_n)\}$
$\{R\}$

we can verify it as follows:

1. Show that P is true *after* initialization
   (which implies that P will be true *before* the loop begins):
   - i.e., show $Q\ \Rightarrow\ wp($initialization$, P)$
2. Show that P is a loop invariant:
   - i.e., for each $i : 1 \leq i < n+1$, show $\{P \wedge B_i\}S_i\{P\}$
   - i.e., show $\{P \wedge B_1\}S_1\{P\}$
     $\ldots$
     $\{P \wedge B_n\}S_n\{P\}$
3. Show that, *if & when* the loop halts, R is true:
   - i.e., show $P \wedge \neg(B_1 \vee \ldots \vee B_n) \Rightarrow R$
4. Show that the loop halts:[1]
   (a) Show that, while still in the loop, t is bounded from below:
      - i.e., show $P \wedge (B_1 \vee \ldots \vee B_n) \Rightarrow t > 0$

   (b) Show that each iteration decrements t
      (i.e., show that each S_i decrements t):[2]
      - i.e., for each $i : 1 \leq i < n+1$, show $\{P \wedge B_i\}\ t1 := t; S_i\ \{t < t1\}$
      - i.e., show $\{P \wedge B_1\}\ t1 := t; S_1\ \{t < t1\}$
        $\ldots$
        $\{P \wedge B_n\}\ t1 := t; S_n\ \{t < t1\}$

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[1]Logically, it makes more sense to do this *before* step 3.
[2]Logically, it makes more sense to do this *before* step 4a.