

CLAUSE FORM

Notation: $X \rightarrow Y$ for: Rewrite all occurrences of X as Y .

Algorithm Clause-Form;

Input A wff α of FOL (or propositional logic);

Output A logically equivalent formula in clause form;

begin

1. Convert α to a logically equivalent formula in Prenex Normal Form:

(a) $(\alpha \equiv \beta) \rightarrow ((\alpha \supset \beta) \wedge (\beta \supset \alpha))$

(b) $(\alpha \supset \beta) \rightarrow (\neg \alpha \vee \beta)$

(c) **repeat:**

i. $\neg \neg \alpha \rightarrow \alpha$

ii. $\neg(\alpha \wedge \beta) \rightarrow (\neg \alpha \vee \neg \beta)$

iii. $\neg(\alpha \vee \beta) \rightarrow (\neg \alpha \wedge \neg \beta)$

iv. $\neg \exists x[\alpha] \rightarrow \forall x[\neg \alpha]$

v. $\neg \forall x[\alpha] \rightarrow \exists x[\neg \alpha]$

until ‘ $\neg$ ’ only applies to atomic wffs;

(d) **begin optional section:**

i. $(\alpha \vee \alpha) \rightarrow \alpha$

ii. $(\alpha \wedge \alpha) \rightarrow \alpha$

end optional section;

(e) Rename variables such that variables bound by different quantifiers have unique names

(f) Move all quantifiers to the left, without changing their order

2. Convert PNF(α) to Skolem Normal Form:

repeat:

(a) $\forall x_1 \dots \forall x_n \exists y[\alpha(y)] \rightarrow \forall x_1 \dots \forall x_n[\alpha(f(x_1 \dots x_n))]$ // f is a new Skolem function

(b) $\exists y[\alpha(y)] \rightarrow \alpha(c)$ // c is a new Skolem constant

until all existential quantifiers are eliminated

3. Convert SNF(α) to Conjunctive Normal Form:

(a) $\forall x[\alpha(x)] \rightarrow \alpha(x)$

(b) **repeat:**

i. $(\alpha \vee (\beta \wedge \gamma)) \rightarrow ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$

ii. **begin optional section:**

A. $(\alpha \vee \alpha) \rightarrow \alpha$

B. $(\alpha \wedge \alpha) \rightarrow \alpha$

end optional section;

until the formula is a conjunction of disjunctions of literals

4. (optionally:) Convert CNF(α) to Clause Form:

(a) $(\alpha \vee \beta) \rightarrow \alpha\beta$ (or: $[\alpha, \beta]$) // $\alpha\beta$ (or: $[\alpha, \beta]$) is a “clause”

(b) $(\alpha \wedge \beta) \rightarrow \{\alpha, \beta\}$ // these are sets of clauses

(c) Rename variables again such that each clause has different variables

end.