CSE 463/563, Knowledge Representation & Reasoning, Spring 2005
FINAL EXAM

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This is a closed-book, closed-notes, closed-neighbor, closed-calculator (or other electronic device), open-mind exam.
The point value of each question is shown in parentheses. Answer all questions in the spaces provided.
There are **??** pages, with 12 questions worth a total of 136 points.

FOR PARTIAL CREDIT, SHOW ALL YOUR WORK. (You may use the backs of pages for scratch work.)

> **PLEASE CIRCLE "463" OR "563", AS APPROPRIATE, ABOVE.**
> **Please put your name on EACH page.**
> **Read the whole test through before beginning, so you can plan your time.**

propositional-logic semantics: 1. _____ (6 points)

propositional-logic proof-theory: 2. _____ (6 points)

propositional-logic resolution: 3. _____ (6 points)

clause form: 4. _____ (12 points)

resolution: 5. _____ (7 points)

unification: 6. _____ (24 points)

FOL representation: 7. _____ (12 points)

FOL representation: 8. _____ (12 points)

FOL clause form: 9. _____ (12 points)

FOL reasoning: 10. _____ (12 points)

SNePS representation: 11. _____ (24 points)

reasoning: 12. _____ (3 points)

Total: _____ (136 points)

Letter-Grade Equivalents:

	CSE463	both	CSE563
A		129–136	
A–		122–128	
B+		114–121	
B		107–113	
B–		99–106	
C+		92–98	
C	77–91		46–91
C–	61–76		
D+	46–60		
D		24–45	
F		0–23	

Note: This version of the exam, with answers, omits the Ziggy cartoon and the 2 appendices.

1. (6 points)

   Use truth tables to show semantically that

   $$\{ (P \supset Q),\ (Q \supset R) \} \models (P \supset R)$$

   Don't just construct a truth table! You must explain *how* the truth table shows this.

	P	Q	R	premise $P \supset Q$	premise $Q \supset R$	conclusion $P \supset R$
1	T	T	T	**T**	**T**	**T**
2	T	T	F	T	F	F
3	T	F	T	F	T	T
4	T	F	F	F	T	F
5	F	T	T	**T**	**T**	**T**
6	F	T	F	T	F	T
7	F	F	T	**T**	**T**	**T**
8	F	F	F	**T**	**T**	**T**

   Both premises are only true on lines 1,5,7,8.
   On all of those lines, the conclusion is also true.

2. (6 points)

Use the rules of inference for our system of natural deduction for propositional logic (see page **??** for specifications of these rules) to show syntactically that

$$\{ (P \supset Q), (Q \supset R) \} \vdash (P \supset R)$$

1.	$P \supset Q$	: assumption
2.	$Q \supset R$	: assumption
*3.	P	: temporary assumption
*4.	$P \supset Q$	: send 1
*5.	Q	: 4,3, $\supset$Elim
*6.	$Q \supset R$	: send 2
*7.	R	: 6,5, $\supset$Elim
*8.	$P \supset R$	: 3,7, $\supset$Intro
9.	$P \supset R$	: return 8

3. (6 points)

   Use Resolution and Refutation to show syntactically that

   $$\{ (P \supset Q),\ (Q \supset R)\ \} \vdash (P \supset R)$$

   ClauseForm$(P \supset Q) = [\neg P, Q]$
   ClauseForm$(Q \supset R) = [\neg Q, R]$
   ClauseForm$(\neg(P \supset R)) = \neg(\neg P \vee R) = (P \wedge \neg R) = [P], [\neg R]$

   In natural-deduction-style notation (other answers are possible):

   1. $[\neg P, Q]$
   2. $[\neg Q, R]$
   3. $[P]$
   4. $[\neg R]$
   5. $[\neg Q]$: 2,4, Resolution
   6. $[\neg P]$: 1,5, Resolution
   7. $[\ ]$: 3,6, Resolution

4. (6 points each; total = 12 points)

Convert the following wffs to clause form:

(a) $((\neg P \supset \neg Q) \wedge \neg P) \supset \neg Q$

$$\neg((\neg\neg P \vee \neg Q) \wedge \neg P) \vee \neg Q$$
$$\neg((P \vee \neg Q) \wedge \neg P) \vee \neg Q$$
$$\neg(P \vee \neg Q) \vee P \vee \neg Q$$
$$(\neg P \wedge Q) \vee (P \vee \neg Q)$$
$$(\neg P \vee P \vee \neg Q) \wedge (Q \vee P \vee \neg Q)$$
$$\{[\neg\mathbf{P}, \mathbf{P}, \neg\mathbf{Q}], [\mathbf{Q}, \mathbf{P}, \neg\mathbf{Q}]\}$$

(b) $\forall x \exists y \forall z \exists w.P(x, y, z, w)$

$$\forall x \forall z \exists w.P(x, f(x), z, w)$$
$$\forall x \forall z.P(x, f(x), z, g(x, z))$$
$$[\mathbf{P}(\mathbf{x}, \mathbf{f}(\mathbf{x}), \mathbf{z}, \mathbf{g}(\mathbf{x}, \mathbf{z}))]$$

5. (7 points total)

   (a) (1 point)

   Can Resolution be used to infer R from $[P, Q]$ and $[\neg P, \neg Q, R]$?

   Circle your answer: **NO**

   ___

   (b) (3 points for truth table; 3 points for explanation; total = 6 points)

   Justify your answer with a truth table, and *explain in English* what the truth table shows about whether R can be inferred from $[P, Q]$ and $[\neg P, \neg Q, R]$.

		conc			prem	prem
P	Q	R	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \vee \neg Q \vee R$
T	T	T	F	F	T	T
T	T	F	F	F	T	F
T	F	T	F	T	T	T
T	F	**F**	F	T	**T**	**T**
F	T	T	T	F	T	T
F	T	**F**	T	F	**T**	**T**
F	F	T	T	T	F	T
F	F	F	T	T	F	T

   There are 2 rows with T premises but F conclusion;
   $\therefore$ premises $\nvdash$ conclusion

6. (6 points each; total = 24 points)

For each of the following pairs of expressions, if they can be unified, show **both** a most general unifier **and** the common instance (i.e., the member of the singleton set W), **else** state *why* they cannot be unified. Let P, Q, be predicates; let f, g be function symbols; let x, y, and z be variables, and let a, b, c, d be constants. Use the notation $\{variable_1/term_1, \ldots, variable_n/term_n\}$ to represent the substitution of the $term_i$s for the $variable_i$s.

(a) $P(y,b,f(y))$ and $P(a,x,z)$

$DS = \{y,a\}, \theta = \{y/a\}, W = \{P(a,b,f(a)),P(a,x,z)\}$
$DS = \{b,x\}, \theta = \{y/a,x/b\}, W = \{P(a,b,f(a)),P(a,b,z)\}$
$DS = \{f(a),z\}, \theta = \{\mathbf{y/a,x/b,z/f(a)}\}, \mathbf{W = \{P(a,b,f(a))\}}$

(b) $Q(c,d,y)$ and $Q(z,x,c)$

$DS = \{c,z\}, \theta = \{z/c\}, W = \{Q(c,d,y),Q(c,x,c)\}$
$DS = \{d,x\}, \theta = \{z/c,x/d\}, W = \{Q(c,d,y),Q(c,d,c)\}$
$DS = \{y,c\}, \theta = \{\mathbf{z/c,x/d,y/c}\}, \mathbf{W = \{Q(c,d,c)\}}$

(c) $P(f(y),y)$ and $P(x,g(x))$

$DS = \{f(y),x\}, \theta = \{x/f(y)\}, W = \{P(f(y),y),P(f(y),g(f(y)))\}$
$DS = \{y,g(f(y))\}, y$ **occurs in** $g(f(y))$**, so occurs check fails, so not unifiable**

(d) $Q(f(c),y)$ and $Q(a,f(b))$

$DS = \{f(c),a\},$ **no variable occurs in a term, so occurs check fails, so not unifiable**

7. (6 points each; total = 12 points)

Consider the following syntax and semantics for the predicates of a language for FOL:

[[[Place(x)]]] = [[x]] is a place
[[[Thing(x)]]] = [[x]] is a thing
[[[For(x,y)]]] = [[x]] is for [[y]]

(a) There is a place for every thing.

$$\forall x[\text{Thing}(x) \supset \exists y[\text{Place}(y) \wedge \text{For}(y,x)]]$$

or

$$\forall x \exists y[\text{Thing}(x) \supset (\text{Place}(y) \wedge \text{For}(y,x))]$$

(b) There is a thing for every place.

$$\forall y[\text{Place}(y) \supset \exists x[\text{Thing}(x) \wedge \text{For}(x,y)]]$$

or other similar wffs.

Note that as a knowledge engineer or logician, your job is to represent the underlying logical form or meaning of an English sentence, not its surface structure. Here, especially given the context provided by the cartoon, the meaning if the first sentence is "for every thing, there is a place for it", not "there is a special place that everything is located in"; hence, the universal quantifier comes first, not the existential.

8. (3 points each; total = 12 points)

Consider the following syntax and semantics for the predicates of a language for FOL:

$[[B(x)]]$	$=$	$[[x]]$ is a baby	$[[L(x)]]$	$=$	$[[x]]$ is logical (or: a logical person).
$[[D(x)]]$	$=$	$[[x]]$ is despised	$[[M(x)]]$	$=$	$[[x]]$ can manage a crocodile.

Represent each of the following sentences (from an example by Lewis Carroll) in this language:

(a) All babies are illogical.

$$\forall x[B(x) \supset \neg L(x)]$$

(b) Nobody is despised who can manage a crocodile.
(I.e., anyone who can manage a crocodile is not despised.)

$$\forall y[M(y) \supset \neg D(y)]$$

or

$$\neg \exists y[D(y) \wedge M(y)]$$

Note, by the way, that the variables eventually need to be different, according to the clause-form algorithm, so you might as well make them different now. I did not take off points for not having different variables, however.

(c) Illogical persons are despised.

$$\forall z[\neg L(z) \supset D(z)]$$

(d) No baby can manage a crocodile.

$$\forall w[B(w) \supset \neg M(w)]$$

or

$$\neg \exists w[B(w) \wedge M(w)]$$

9. (3 points each; total = 12 points)

Translate each of 8a–8c and the **negation** of 8d into clause form:

(a)
$$[\neg B(x), \neg L(x)]$$

(b)
$$[\neg M(x), \neg D(x)]$$

(c)
$$[L(x), D(x)]$$

(d)
$$\neg \forall w[B(w) \supset \neg M(w)]$$

—>

$$\exists w.\neg[B(w) \supset \neg M(w)]$$

—>

$$\exists w.\neg[\neg B(w) \vee \neg M(w)]$$

—>

$$\exists w[B(w) \wedge M(w)]$$

—>

$$\{[\mathbf{B}(\mathbf{c})], [\mathbf{M}(\mathbf{c})]\}$$

where c is a Skolem constant

10. (12 points)

Using Resolution, Unification, and Refutation, and showing MGUs, show that 8d can be derived from 8a–8c.

1. $[\neg B(x), \neg L(x)]$
2. $[\neg M(y), \neg D(y)]$
3. $[L(z), D(z)]$
4. $[B(c)]$
5. $[M(c)]$
6. $[\neg D(c)]$: 2,5, Resolution, with MGU = $\{y/c\}$
7. $[L(c)]$: 3,6, Resolution, with MGU = $\{z/c\}$
8. $[\neg B(c)]$: 1,7, Resolution, with MGU = $\{x/c\}$
9. [ ]   : 4,8, Resolution

Other answers are possible.

11. (6 points each; total = 24 points)

Represent the following text passage in a **single** (i.e., fully interconnected) SNePS network, using **only** the standard case frames given on p. **??** (i.e., no "lex" arcs or "propername" arcs are needed). If you need to provide any new case frames, give their syntax and semantics. (Hint: Since SNePS has no existential quantifier, you can interpret "some *A*s are *B*s" as if it were in Skolem Normal Form: "a particular *A* is a *B*".)

(a) Mary believes that all birds can fly.

In SNePSUL (run this in SNePS, and use the "show" command to see what the networks look like):

```
(assert agent Mary
        act (build action believe
                   object (build forall $x
                                 ant (build member *x
                                            class bird)
                                 cq  (build object *x
                                            property fly))))
```

(b) She (also) believes that all birds are mammals.

```
(assert agent Mary
        act (build action believe
                   object (build forall $x
                                 ant (build member *x
                                            class bird)
                                 cq  (build member *x
                                            class mammal))))
```

or

```
(assert agent Mary
        act (build action believe
                   object (build subclass bird
                                 superclass mammal)))
```

(c) (But) it's not the case that all birds can fly.

```
(assert min 0 max 0
        arg (find forall *x
                  ant (build member *x
                             class bird)
                  cq  (build object *x
                             property fly)))
```

I.e., assert an "andor 0 0" pointing to the object of Mary's first belief.

(d) (Even though) it is the case that they are mammals. Note that this is false, but we can still represent that Cassie believes it! There are 2 ways to do this: First, you could use an "assert" instead of a "build" in part (b) above:

```
(assert agent Mary
        act (build action believe
                   object (ASSERT forall $x
                                  ant (build member *x
                                             class bird)
                                  cq  (build member *x
                                             class mammal))))
```

Second, you can assert an "andor 1 1" pointing to that node:

```
(assert min 1 max 1
        arg (find forall *x
                   ant (build member *x
                              class bird)
                   cq  (build member *x
                              class mammal)))
```

12. (3 points)

WARNING: This problem is trickier than it looks. Do not even attempt it till you have finished with all the other questions. Do not *guess* at the answer! *Compute* the answer instead, using techniques you have learned this semester. There is a long, hard way to do it, and a quick, easy way. No matter which way you do it, it is only worth 3 points!

Both of the following are true about a particular hand of cards (read them carefully!):

(a) If there is a Jack in the hand, then there is a King in the hand, or else if there is no Jack in the hand, then there is a King in the hand.

(b) There is a Jack in the hand.

Represent this in the language of propositional logic (be sure to provide the syntax and semantics of your language), and then use an appropriate method of reasoning (either syntactic or semantic) to answer the following question (and to justify your answer):

(c)*Must* there be a King in the hand?

There are 2 ways to interpret (a): Interpreting "or else" as exclusive-or and interpreting it as inclusive-or. This is a famous "logical illusion", because, in fact, most people—without thinking—actually interpret it as **conjunction**! They then conclude, erroneously, that there must be a King in the hand. A few of you did that.

　　　Most of you interpreted it as inclusive-or. But then most of those of you who did that went on to make a **reasoning error**! If you interpret (a) inclusively, it is a tautology (do the truth table, but don't make any mistakes!)! Since it is a tautology, it conveys no information about the actual hand. Proposition (b) also says nothing about Kings. So, on this interpretation, it might be the case that there's a King in the hand, and it might not be the case that there's a King in the hand. So, it is not the case that there **must** be a King in the hand!

　　　A few of you interpreted it as exclusive-or (which is actually what "or else" is intended to mean, by the way!). In this case, believe it or not, there **cannot** be a King in the hand! I urge you to try this again. Either do a truth table (but be careful!) or convert to clause form (a bit harder). For more information on this, go to:

http://www.cse.buffalo.edu/~rapaport/575/F01/logical.illusion.html
and then to:
http://www.cse.buffalo.edu/~rapaport/mental-models.html