

**Homework #9**  
**FOL: Automated Theorem Proving**

**Last Update: 1 April 2005**

1. For each of the following sets of pairs of atomic wffs, compute a most general unifier if one exists. If none exists, then state why not (e.g., state which part of the unification algorithm fails).
  - (a)  $\{Q(c, b, c), Q(y, z, x)\}$
  - (b)  $\{R(h(b, c), z), R(z, h(y, y))\}$
  - (c)  $\{Younger(son-of(x), x), Younger(son-of(y), bill)\}$  (where 'bill' is a constant)
  - (d)  $\{Knows(son-of(x), x), Knows(y, y)\}$
  
2. From "Horses are animals", it follows that "The head of a horse is the head of an animal." (This was how Aristotle showed that classical logical "syllogisms" (inferences) were not sufficient for doing all of logic; you need FOL to handle this problem.) Demonstrate that this inference is valid by carrying out the following steps:
  - (a) Translate the premise and the conclusion into the language of FOL. Use 3 predicates, whose syntax and semantics are shown below:
 
$$[[Animal(x)]] = [[x]] \text{ is an animal}$$

$$[[Horse(x)]] = [[x]] \text{ is a horse}$$

$$[[Head(x, y)]] = [[x]] \text{ is the head of } y$$
  - (b) Negate the conclusion, and convert the premise and the negated conclusion to clause form, using the algorithm presented in lecture.
  - (c) Use resolution and refutation to show that the conclusion follows from the premise. Be sure to show any MGUs that you might need.

3. (a) Using the following syntax and semantics, translate the following English sentences (adapted from: Carroll, Lewis (1896), *Symbolic Logic*, reprinted in: Lewis Carroll, *Symbolic Logic and the Game of Logic (both books bound as one)* (New York: Dover, 1958)) into the language of FOL:

$[[d]]$  = this dish  
 $[[N(x)]]$  =  $[[x]]$  is nice  
 $[[P(x)]]$  =  $[[x]]$  is a pudding  
 $[[W(x)]]$  =  $[[x]]$  is wholesome

- i. All puddings are nice.
  - ii. This dish is a pudding.
  - iii. No nice things are wholesome.
  - iv. (Therefore:) This dish is unwholesome.
- (b) Convert each of your FOL translations of 3(a)i–3(a)iii and the *negation* of 3(a)iv into clause form, using our algorithm from lecture.
- (c) Use resolution, unification, and refutation to show that the inference is valid. Be sure to show an MGU for each application of resolution.

4. Here are 2 wffs in the language of FOL:

(A)  $\forall x \exists y [x \geq y]$

(B)  $\exists y \forall x [x \geq y]$

- (a) Let the domain of interpretation be all natural numbers: 0, 1, 2, 3, . . . , and let  $[[x \geq y]]$  = “ $[[x]]$  is greater than or equal to  $[[y]]$ ”. Under this interpretation, translate (A) and (B) into English.
- (b) Is (A) true under this interpretation? Why?
- (c) Is (B) true under this interpretation? Why?
- (d) Using resolution, try to prove that (A) follows from (B). Do this even if you think that (B) does not logically entail (A); continue until the proof breaks down and you cannot proceed (if it does break down). Show the unifying substitution for each resolution step. If the proof fails, explain exactly where, how, and why it breaks down.
- (e) In the same way, try to prove that (B) follows from (A).

**DUE: AT THE BEGINNING OF LECTURE, FRIDAY, APRIL 8**