CSE 463/563, Knowledge Representation & Reasoning, Spring 2005
MIDTERM EXAM

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This is a closed-book, closed-notes, closed-neighbor, closed-calculator (and other electronic devices), open-mind exam. The point value of each question is shown in parentheses. Answer all questions in the spaces provided. There are 12 pages, with 7 questions worth a total of 102 points.

FOR PARTIAL CREDIT, SHOW ALL YOUR WORK. (You may use the backs of pages for scratch work.)

PLEASE CIRCLE "463" OR "563", AS APPROPRIATE, ABOVE.

Please put your name on EACH page.

Read the whole test through before beginning, so you can plan your time.

1. _____ (12 points)

2. _____ (12 points)

3. _____ (12 points)

4. _____ (6 points)

5. _____ (18 points)

6. _____ (36 points)

7. _____ (6 points)

Total: _____ (102 points)

Letter-Grade Equivalents:

	463	both	563
A		97–102	
A–		92– 96	
B+		86– 91	
B		80– 85	
B–		75– 79	
C+		69– 74	
C	58–68		35– 68
C–	46–57		
D+	35– 45		
D		18– 34	
F		0– 17	

1. (6 points for syntax/semantics, 6 points for proof; 12 points total)

   Using our language for propositional logic, <u>represent</u> (and give the syntax and semantics of your representation) and <u>prove</u> **syntactically**, using <u>only</u> the rules of inference found at the end of this exam, that:

   > {If Block A is on the table, then Block A is not on Block B., If Block A is not on Block B, then Block B is clear.} ⊢ If Block A is on the table, then Block B is clear.

[[A]] = Block A is on the table.
[[B]] = Block A is on block B.
[[C]] = Block B is clear.

1.	A ⊃ B	: assumption
2.	¬B ⊃ C	: assumption
*3.	A	: temporary assumption
*4.	A ⊃ ¬B	: send 1
*5.	¬B	: 4,3,⊃Elim
*6.	¬B ⊃ C	: send 2
*7.	C	: 6,5,⊃Elim
*8.	A ⊃ C	: 3,7,⊃Intro
9.	A ⊃ C	: return 8

2. (6 points for truth table, 6 points for explanation; 12 points total)

Use a truth table to show that:

$$\{(P \supset Q), (Q \supset R)\} \models (P \supset R)$$

Do not just compute a truth table. Explain in English how the truth table shows this.

	P	Q	R	premise $P \supset Q$	premise $Q \supset R$	conclusion $P \supset R$
1	T	T	T	**T**	**T**	**T**
2	T	T	F	T	F	F
3	T	F	T	F	T	T
4	T	F	F	F	T	F
5	F	T	T	**T**	**T**	**T**
6	F	T	F	T	F	T
7	F	F	T	**T**	**T**	**T**
8	F	F	F	**T**	**T**	**T**

Both premises are only true on lines 1,5,7,8.
On all of those lines, the conclusion is also true.

3. (6 points for truth table, 6 points for explanation; 12 points total)

Use a truth table to determine whether:

$$\{(P \supset (Q \vee R)), \neg Q\} \models (P \vee R)$$

Again, explain your answer in English.

	P	Q	R	Q∨R	premise P⊃(Q∨R)	premise ¬Q	conclusion P∨R
1	T	T	T	T	T	F	T
2	T	T	F	T	T	F	T
3	T	F	T	T	T	T	T
4	T	F	F	F	F	T	T
5	F	T	T	T	T	F	T
6	F	T	F	T	T	F	F
7	F	F	T	T	T	T	T
8	F	F	F	F	**T**	**T**	**F**

On line 8, the premises are true, but the conclusion is false.
So, the inference is invalid.

4. (6 points)

Find a wff of propositional logic (other than one of the premises!) that **does** follow from:

$$\{(P \supset (Q \vee R)), \neg Q\}$$

and then prove **syntactically** that it follows.

Answers may vary!

1.	$P \supset (Q \vee R)$	: assumption
2.	$\neg Q$	: assumption

{Show P ⊃ R:}

*3.	P	: temporary assumption
*4.	$P \supset (Q \vee R)$	: send 1
*5.	$(Q \vee R)$	: 4,3,⊃Elim
*6.	$\neg Q$	: send 2
*7.	R	: 5,6,∨Elim
*8.	$P \supset R$	: 3,8,⊃Intro
9.	$P \supset R$	: return 8

5. (6 points each; 18 points total)

Using our language for FOL and the following syntax and semantics, represent the following sentences inspired by this cartoon:

[[ugly]] = "Our website is ugly"
[[Person(x)]] = x is a person on Earth
[[Says(x,y)]] = x says y

(a) Everyone says our web site is ugly.

$$\forall x.\text{Says}(x, \text{ugly})$$

(b) Every person on Earth says our web site is ugly.

$$\forall x[\text{Person}(x) \supset \text{Says}(x, \text{ugly})]$$

(c) Just one person on Earth says our web site is ugly.

$$\exists x[\text{Person}(x) \wedge \text{Says}(x, \text{ugly}) \wedge \forall y[(\text{Person}(y) \wedge \text{Says}(y, \text{ugly})) \supset y = x]]$$

6. (6 points each; 36 points total)

   Using our language for FOL and the syntax and semantics on page 11, represent the following sentences:

   (a) "Some cowboys don't have guns, but all the cowboys have boots and hats on." (Michael Rapaport, age 3)

   $$\exists x[\text{Cowboy}(x) \wedge \neg\text{Gun}(x)] \wedge \forall y[\text{Cowboy}(y) \supset (\text{Boots}(y) \wedge \text{Hat}(y))]$$

   (b) "All menu items are not always on display." (From an actual menu.)

   $$\neg\forall x[\text{Menu}(x) \supset \text{Display}(x)]$$

   (c) "All the Great Lakes won't freeze, but all is not well." (Heard on NPR.)

   $$\neg\forall x[\text{GreatLake}(x) \supset \text{Freeze}(x)] \wedge \neg\forall x.\text{Well}(x)$$

(d) "Some bugs are dead. And some are alive." (Michael Rapaport, age 3)

$$\exists x[\text{Bug}(x) \wedge \text{Dead}(x)] \wedge \exists y[\text{Bug}(y) \wedge \text{Alive}(y)]$$

(e) "When someone ropes someone, that someone can't move." (Michael Rapaport, age 3)

$$\forall x \forall y[\text{Ropes}(x,y) \supset \neg \text{Move}(y)]$$

(f) "The United States is a country. So, a map of the United States is a map of a country!" (Michael Rapaport, age 6)

(Note: There are at least two different ways you can represent this, depending on whether you interpret 'map' as a relation or a function; only represent in one way, whichever you prefer.)

   i.

$$\text{Country}(\text{us}) \vdash \forall x[\text{Map}(x,\text{us}) \supset \exists y[\text{Country}(y) \wedge \text{Map}(x,y)]]$$

   ii.

$$\text{Country}(\text{us}) \vdash \forall x[x = \text{map}(\text{us}) \supset \exists y[\text{Country}(y) \wedge x = \text{map}(y)]]$$

(Note: Instead of $\vdash$, you could also have used $\supset$. The translation issue then turns on whether 'So' in the English sentence means "therefore", or "(if) then". By the Deduction Theorem, these are equivalent.)

7. (3 points each; 6 points total)

Using our language for FOL and the following syntax and semantics, represent the following sentences inspired by this cartoon. **Warning:** This is trickier than it looks!

[[Snowflake(x)]] = x is a snowflake.

(a) No two snowflakes are the same.

$$\forall x \forall y [(\text{Snowflake}(x) \land \text{Snowflake}(y)) \supset \neg(x = y)]$$

Note that this assumes that x and y are never assigned the same values, an assumption that FOL normally does not make. For more on this, see Shapiro, Stuart C. (1986), "Symmetric Relations, Intensional Individuals, and Variable Binding", *Proceedings of the IEEE* 74(10): 1354–1363 [http://www.cse.buffalo.edu/~shapiro/Papers/uvbr.pdf].

The following wff, which is arguably more correct, is also problematic, but acceptable:

$$\forall x \forall y [(\text{Snowflake}(x) \land \text{Snowflake}(y) \land \neg(x = y)) \supset \neg(x = y)]$$

The first occurrence of "$\neg(x = y)$" (in the antecedent) represents that there are **two** snowflakes. The occurrence in the consequent represents that they are distinct. But that's redundant! In fact, this is a tautology, and it is rarely correct to translate an English sentence into an FOL wff that is a tautology. In this case, however, that's what we seem to have!

(b) No two things are the same.

Here, the issues are even clearer:

$$\forall x \forall y . \neg(x = y)$$

(assuming unique variable binding, as discussed above), or:

$$\forall x \forall y [\neg (x = y)) \supset \neg (x = y)]$$

(which is just a tautology again).

SYNTAX AND SEMANTICS FOR PROBLEM 6

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Constants:
[[us]] = the United States

1-Place Function Symbols:
[[map(x)]] = a map of x

1-Place Predicates:
[[Alive(x)]] = x is alive.
[[Boots(x)]] = x has boots on.
[[Bug(x)]] = x is a bug.
[[Country(x)]] = x is a country.
[[Cowboy(x)]] = x is a cowboy.
[[Dead(x)]] = x is dead.
[[Display(x)]] = x is always on display.
[[Freeze(x)]] = x will freeze.
[[GreatLake(x)]] = x is a Great Lake.
[[Gun(x)]] = x has a gun.
[[Hat(x)]] = x has a hat on.
[[Menu(x)]] = x is a menu item.
[[Move(x)]] = x can move.
[[Well(x)]] = x is well.

2-Place Predicates:
[[Map(x,y)]] = x is a map of y.
[[Ropes(x,y)]] = x ropes y.

RULES OF INFERENCE

You can tear this page off.

¬Intro: If, from an assumption α, you can infer both β and ¬β in a subproof
           Then you may infer ¬α in that subproof

¬Elim: If, from an assumption ¬α, you can infer both β and ¬β in a subproof
           Then you may infer α in that subproof

∨Intro: From α (or from β)
           Infer (α ∨ β)

∨Elim: From (α ∨ β)
           & ¬α (or ¬β)
           Infer β (or α, respectively)

⊃Intro: If, from an assumption α, you can infer β in a subproof
           Then you may infer (α ⊃ β) in that subproof

⊃Elim: From (α ⊃ β)
           & α
           Infer β

Send: If α appears as the only sentence on a line *L* of a proof
           Then you may send α into any subproof that is immediately subordinate to the proof at *L*

Return: If α has been inferred by ¬Intro, ¬Elim, or ⊃Intro as the last line in any subproof
           Then you may return α as an inference to that subproof's immediately superordinate proof