Homework #1 Answers

Note: There were many permissible variations on the wording of "translation"-type answers. We have taken time just to comment on a couple of them. Apart from these comments, the key represents that answers need not be verbose in order to be fine. It was written mostly by the TA's.

1 1.1.10.

p = "The election is decided"
q = "The votes have been counted"
(a) $\neg p$ The election is not decided.
(b) $p \vee q$ The election is decided or the votes have been counted.
(c) $\neg p \wedge q$ The election is not decided and the votes have been counted.
(d) $q \rightarrow p$ If the votes have been counted, then the election is decided.
(e) $\neg q \rightarrow \neg p$ If the votes have not been counted, then the election is not decided.
(f) $\neg p \rightarrow \neg q$ If the election is not decided, then the votes have not been counted.
(g) $p \leftrightarrow q$ The election is decided if and only if the votes have been counted.
(h) $\neg q \vee (\neg p \wedge q)$ The votes have not been counted or the election is not decided and the votes have been counted.

2 1.1.12.

p : You have the flu.
q : You miss the final examination.
r : You pass the course.
(e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ If you have the flu, then you will not pass the course, or if you miss the final examination, then you will not pass the course.
Simplified:
If you have the flu and you miss the final examination, then you will not pass the course.

(Notice that it should be AND instead of OR. This example was covered in lecture with Buffalo Bills happenings, except with $R =$ Bills Win in place of $\neg r$. Another way of interpreting it is to reason that it says: either there is a causal implication from having the flu to not passing the course, or there is a causal implication from missing the final to not passing. You don't know which—it could be both—but one of them holds. Hence if you both have the flu and miss the final, you will trigger one of them and not pass. It is still weird that the equivalence goes the other way. Suppose you are in a situation where having the flu and missing the final will cause you to fail. That assessment is invalid only in the case where p, q, r are all true (passing rather than getting an I, F, or withdrawal). The first implication is invalid only when p and r are both true, and the second is invalid only when q and r are true. Hence both are invalid when and only when all of p, q, r are true. Both being invalid means that the OR of the two implications is invalid. Hence the assessment of (e) is semantically invalid also only in the all-true case. Thus (e) and the "simplified" version are valid in the same outcome cases, so they are equivalent logically. This flies in the face of the idea that only the combination of flu and playing hooky might cause you to fail, but...)

3 1.1.13.

p : You drive over 65 miles per hour.
q : You get a speeding ticket.
(a) $\neg p$
(b) $p \wedge \neg q$
(c) $p \rightarrow q$
(d) $\neg p \rightarrow \neg q$
(e) $p \rightarrow q$
(f) $q \wedge \neg p$
(g) $q \rightarrow p$
If you interpreted "whenever" differently than the book, a concise explanation is necessary.

4 1.1.24

(a) Acceptable: If I will remember to send you the address, then you send me an email message.
Better Answer: If I remember to send you the address, then it must have been because you sent me an e-mail message.
(b) If you were born in the United States, then you are a citizen of this country.
(c) If you keep your textbook, then it will be a useful reference in your future courses.
(d) If their goalie plays well, then the Red Wings will win the Stanley Cup.

5 1.1.26

(a) You will get an A in this course if and only if you learn how to solve discrete mathematics problems.
(b) You read the newspaper every day if and only if you are in-formed.
(c) It rains if and only if it is a weekend day.
(d) You can see the wizard if and only if the wizard is not in.

6 1.3.9

(b) $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

(c) $\neg p \rightarrow (p \rightarrow q)$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow (p \rightarrow q)$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	0	1	1

(e) $\neg(p \rightarrow q) \rightarrow p$

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	1
1	1	1	0	1

(f) $\neg(p \rightarrow q) \rightarrow \neg q$

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
0	0	1	0	1	1
0	1	1	0	0	1
1	0	0	1	1	1
1	1	1	0	0	1

These tables have helper columns for every subformula; it was OK to skip a couple. The "right half" of (e) was just a variable—the remark on the problem set came from a momentary confusion of "page 35" with "problem 35" on page 15, which I have assigned in other years.