CSE191 Assignment 2 Answer Key Fall 2013

1 1.3.8.

(c) p = "James is young"
q = "James is strong"
We can represent the given english statment as $p \wedge q$.
The negation can be represented as: $\neg(p \wedge q) \equiv \neg p \vee \neg q$ [Apply DeMorgan's Law].
The negation can be expressed in english as: James is not young or James is not strong.

(d) p = "Rita will move to Oregon"
q = "Rita will move to Washington"
We can represent the given english statment as $p \vee q$.
The negation can be represented as: $\neg(p \vee q) \equiv \neg p \wedge \neg q$ [Apply DeMorgan's Law].
The negation can be expressed in english as: Rita will not move to Oregon and Rita will not move to Washington.

2 1.3.10.

(b)

p	q	r	$p \to q$	$q \to r$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
0	0	0	1	1	1	1
1	0	0	0	1	0	1
0	1	0	1	0	1	1
1	1	0	1	0	0	1
0	0	1	1	1	1	1
1	0	1	0	1	1	1
0	1	1	1	1	1	1
1	1	1	1	1	1	1

Explanation: If p implies q and q implies r, then p should imply r because implication is transitive. In other words, if you have a chain of conditions where each condition causes the following condition, then the first condition should cause the last condition.

(d)

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
0	0	0	0	1	1	1
1	0	0	1	0	1	1
0	1	0	1	1	0	1
1	1	0	1	0	0	1
0	0	1	0	1	1	1
1	0	1	1	1	1	1
0	1	1	1	1	1	1
1	1	1	1	1	1	1

Explanation: If p implies r, q implies p, and either p or q is true, then r is true. This follows because we can pick which of p or q is true and this variable must imply r since both p and q imply r. Therefore, r must be true.

There are three valid ways to do the next kind of problems: (1) truth table, (2) use equivalences, or (3) show they are true for exactly the same assignments. The latter two are harder but can save considerable work.

3 1.3.16.

Method (3): $p \leftrightarrow q$ is true exactly when p and q are both true or p and q are both false which is represented by $(p \wedge q) \vee (\neg p \wedge \neg q)$.

1.3.20

Method (1):

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$
0	0	0	1	1
1	0	1	0	0
0	1	1	0	0
1	1	0	1	1

1.3.22

Method (2):
$(p \rightarrow q) \wedge (p \rightarrow r)$
$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ [Convert $\rightarrow$ to $\vee$]
$\equiv \neg p \vee (q \wedge r)$ [Distributivity of $\vee$ over $\wedge$]
$\equiv p \rightarrow (q \wedge r)$ [Convert $\vee$ to $\rightarrow$]

1.3.24

Method (2):
$(p \rightarrow q) \vee (p \rightarrow r)$
$\equiv (\neg p \vee q) \vee (\neg p \vee r)$ [Convert $\rightarrow$ to $\vee$]
$\equiv (\neg p \vee \neg p) \vee (q \vee r)$ [Associativity and Communitivity of $\vee$]
$\equiv \neg p \vee (q \vee r)$ [$\neg p \vee \neg p \equiv \neg p$]
$\equiv p \rightarrow (q \vee r)$ [Convert $\vee$ to $\rightarrow$]

4 NOR Problem

It suffices to show that $\neg p$ and $p \wedge q$ are equivalent to propositional formulas whose only operator is NOR.

Claim 1: $\neg p \equiv p$ NOR p.

p	$\neg p$	p NOR p
0	1	1
1	0	0

Claim 2: $p \wedge q \equiv (p$ NOR $p)$ NOR $(q$ NOR $q)$.

p	q	$p \wedge q$	p NOR p	q NOR q	$(p$ NOR $p)$ NOR $(q$ NOR $q)$
0	0	0	1	1	0
1	0	0	0	1	0
0	1	0	1	0	0
1	1	1	0	0	1