

1 Problem 1 (15 points)

Translate each of the following into English, where $H(x)$ is the predicate that x hops, and $R(x)$ is the predicate that x is a rabbit.

(a) $(\forall x)(H(x) \rightarrow R(x))$ is "Every animal that hops is a rabbit."

(b) $(\forall x)(R(x) \vee H(x))$ is "Every animal hops or is a rabbit."

(c) $(\exists x)(H(x) \rightarrow R(x))$ is "There exists an animal such that, if it hops, then it is a rabbit".

(d) $(\exists x)(R(x) \vee H(x))$ is "There exists an animal that hops or is a rabbit."

(e) $(\exists x R(x)) \vee (\exists x H(x))$ is "There exists an animal that hops or there exists an animal that is a rabbit"

Note also that /if/ we take it for granted as "background knowledge" that all rabbits hop, i.e. $(\forall x)(R(x) \rightarrow H(x))$, then statements (b) and (d) are "redundant". But of course, there might be hopeless rabbits out there...

2 Problem 2 (9 points)

(e) $((\exists x)H(x)) \vee ((\exists y)R(y))$ —is that equivalent to (d)?

KWR note: I've renamed the second "x" to "y" here, as is AOK with a "dummy variable" as opposed to a free variable.) This says, "Either there exists an animal that hops, or there is a rabbit." Is this equivalent to (d) now? IMHO yes, it's pretty clear from the language. FYI, a "semantic" proof would go like this: Mindful that there are two separate quantifiers here, the truth of (e) means:

there exist objects a,b in the domain D such that a hops or b is a rabbit.

Whereas the truth of (d) is the seemingly more-demanding statement:

there exists an object a in the domain D such that a hops or a is a rabbit.

Well, for (a,b) given in (d), either a hops or b is a rabbit. If the former, then a witnesses the statement in (d). If the latter, then taking a = b witnesses the statement in (d). Thus (e) \implies (d), and since (d) \implies (e) is clear, they are equivalent.

3 Exercise 1.5.24 (12 points)

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people. Let:

$S(x) = x$ is a student in your class

$C(x) = x$ has a cellular phone

$F(x) = x$ has seen a foreign movie.

$Sw(x) = x$ can swim

$Q(x) = x$ can solve quadratic equations

$R(x) = x$ desires to be rich.

(a) Everyone in your class has a cellular phone.

$(\forall x : student)C(x)$

$(\forall x : people)(S(x) \rightarrow C(x))$

(b) Somebody in your class has seen a foreign movie.

$(\exists x : student)F(x)$

$(\exists x : people)(S(x) \wedge F(x))$

- (c) There is a person in your class who cannot swim.
 $(\exists x : student)\neg Sw(x)$
 $(\exists x : people)(S(x) \wedge \neg Sw(x))$
- (d) All students in your class can solve quadratic equations.
 $(\forall x : student)Q(x)$
 $(\forall x : people)(S(x) \rightarrow Q(x))$
- (e) Some student in your class does not want to be rich.
 $(\exists x : student)\neg R(x)$
 $(\exists x : people)(S(x) \wedge \neg R(x))$
 same as(c)

4 Exercise 1.5.28 (b,c,d,e) (12 points)

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

Let $T(x) = x$ is a tool, $C(x) = x$ is in the correct place, and $E(x) = x$ is in excellent condition.

- (b) All tools are in the correct place and are in excellent condition.

$$(\forall x)(T(x) \rightarrow (C(x) \wedge E(x)))$$

KWR note: This is where I hope my lecture riffs about “going vanilla” help a bit—it’s contrary to the meaning here to say you are using the domain of tools and just writing $(\forall x)(C(x) \wedge E(x))$, The difference is shown by the next question.

- (c) Everything is in the correct place and in excellent condition.

$$(\forall x)(C(x) \wedge E(x))$$

- (d) Nothing is in the correct place and is in excellent condition.

KWR note: *This one* was treated as being potentially ambiguous. The preferred reading is to insert “both” after “nothing is”, and this gives:

$$(\forall x)(\neg(C(x) \wedge E(x)))$$

But it is also possible to read the speaker as intending to say “nothing” also before “is in excellent condition.” This gives:

$$(\forall x)(\neg C(x) \wedge \neg E(x))$$

- (e) One of your tools is not in the correct place, but it is in excellent condition.

$$(\exists x)(T(x) \wedge \neg C(x) \wedge E(x))$$

5 Exercise 1.5.36 (12 points)

Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

Also say which of the false statements become(s) true when the domain is restricted to the nonnegative integers, that is, to \mathbb{N}

- a) $\forall x(x^2 \neq x)$ 0 or 1
 b) $\forall x(x^2 \neq 2)$ $\pm\sqrt{2}$
 c) $\forall x(|x| > 0)$ 0

if the domain is restricted to the nonnegative integers, b) becomes true