CSE191 Assignment 4 Answer Key Fall 2013

1 Exercise 1.5.28 (21)

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
f) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$ False
g) $\forall x \exists y (x + y = 1)$ True
h) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$ False
i) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$ is False because when x=0, y=2 and y=-1. Since y cannot have two different values, $x = 0$ is a counter example.

2 Exercise 1.5.30 (3+3+6=12)

Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
a) $\neg \exists y \exists x P(x, y) \equiv \forall y \forall x \neg P(x, y)$
b) $\neg \forall x \exists y P(x, y) \equiv \exists x \forall y \neg P(x, y)$
c) $\neg \exists y (Q(y) \wedge \forall x \neg R(x, y)) \equiv \forall y \neg (Q(y) \wedge \forall x \neg R(x, y)) \equiv \forall y (\neg Q(y) \vee \neg \forall x \neg R(x, y))$
$\equiv \forall y (\neg Q(y) \vee \exists x R(x, y))$

3 Exercise 1.6.4 (4*3=12)

What rule of inference is used in each of these arguments?
a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
Simplification
b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
Disjunctive syllogism
c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
Modus ponens
d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
Addition

4 Exercise 1.6.10 (4*6=24)

a) One version of the answer could be:
H(x) = I play hockey
S(x) = I am sore the next day
W(x) = I use the whirlpool

Step	Reason
1. $H(x) \rightarrow S(x)$	Premise
2. $S(x) \rightarrow W(x)$	Premise
3. $H(x) \rightarrow W(x)$	Hypothetical syllogism from (1) and (2)
4. $\neg W(x)$	Premise
5. $\neg H(x)$	Modus tollens from (3) and (4)

I do not play hockey
b) One version of the answer could be:

W(x) = I work
S(x) = It is sunny
P(x) = It is partly sunny
[MTF][WSP] = [Monday, Tuesday, Friday] [Work, Sunny, Partly Cloudy]

Step	Reason
1. $W(x) \to S(x) \vee P(x)$	Premise
2. $MW(x) \vee FW(x)$	Premise
3. $\neg TS$	Premise
4. $\neg FP$	Premise
5. $FW \to FS \vee FP$	Universal instantiation of (1)
6. $FW \to FS$	Disjunctive syllogism of (4) and (5)
7. $MW \vee FS$	Modus ponens of (2) and (6)

Work on Monday or Friday is Sunny

Step	**Reason**
1.If I work, it is either sunny or partly sunny.	Premise
2.I worked last Monday or I worked last Friday.	Premise
3.Last Monday or last Friday is either sunny or partly sunny.	Modus ponens using (1) and (2)
4.It was not partly sunny on Friday.	Premise
5.If I worked on Friday, then it was sunny or partially sunny on that day.	Instantiation from (1)
6.If I worked on Friday, then it was sunny on that day.	Disjunctive syllogism using (4), (5)
7.It was not sunny on Tuesday.	Premise
8.If I worked on Tuesday, then it was partially sunny on that day.	Disjuntive syllogism using (1), (7)

e) One version of the answer could be:
x = food, t = tofu, c = cheeseburger
H(x) = x is healthy to eat
G(x) = x tastes good
Y(x) = You eat x

Step	Reason
1. $\forall x H(x) \to \neg G(x)$	Premise
2. $H(t)$	Premise
3. $\forall x Y(x) \to G(x)$	Premise
4. $\neg Y(t)$	Premise
5. $\neg H(c)$	Premise
6. $H(t) \to \neg G(t)$	Universal instantiation of (1)
7. $\neg G(t)$	Modus ponens of (2) and (6)
8. $Y(t) \to G(t)$	Universal instantiation of (3)
9. $\neg Y(t)$	Modus tollens of (7) and (8)

You do not eat Tofu

f) One version of the answer could be:
D(x) = I am dreaming
H(x) = I am hallucinating
E(x) = I see elephants running down the road

Step	Reason
1. $D \vee H$	Premise
2. $\neg D$	Premise
3. $H \to E$	Premise
4. H	Disjunctive syllogism of (1) and (2)
5. E	Modus ponens of (3) and (4)

I see elephants running down the road

5 Exercise 1.6.12 (15)

Using Exercise 11, the argument form with premises $(p \wedge t) \rightarrow (r \vee s), q \rightarrow (u \wedge t), u \rightarrow p$, and$\neg s$ and conclusion $q \rightarrow r$ is valid if the argument form with premises $(p \wedge t) \rightarrow (r \vee s), q \rightarrow (u \wedge t), u \rightarrow p, \neg s$ and q and conclusion r is valid.

Step	**Reason**
1.$q \rightarrow (u \wedge t)$	Premise
2.$u \rightarrow p$	Premise
3.$q \rightarrow u$	Simplification of (1)
4.$q \rightarrow p$	Hypothetical syllogism using (2) and (3)
5.$q \rightarrow t$	Simplification of (1)
6.$q \rightarrow (p \wedge t)$	Conjunction
7.$(p \wedge t) \rightarrow (r \vee s)$	Premise
8.$q \rightarrow (r \vee s)$	Hypothetical syllogism using (6) and (7)
9.q	Premise
10.$r \vee s$	Modus ponens using (8) and (9)
11.$\neg s$	Premise
12.r	Disjunctive syllogism using (10) and (11)

So, the argument form with premises $(p \wedge t) \rightarrow (r \vee s), q \rightarrow (u \wedge t), u \rightarrow p$, and$\neg s$ and conclusion $q \rightarrow r$ is valid.

6 Exercise 1.6.14 (9)

a) One version of the answer could be:

Step	**Reason**
1.Linda, a student in this class, owns a red convertible.	Premise
2.Everyone who owns a red convertible has gotten at least one speeding ticket.	Premise
3.Linda, who owns a red convertible has gotten at least one speeding ticket.	Universal instantiation from (2)
4.Linda is a student in this class.	Simplification of (1)
5.Someone in this class has gotten a speeding ticket.	Existential generalization from (3) and (4)

b)

Step	**Reason**
1.Each of the five roomates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics.	Premise
2.Every student who has taken a course in discrete mathematics can take a course in algorithms.	Premise
3.Each of the five roomates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, who has taken a course in discrete mathematics can take a course in algorithms	Universal instantiatiaon from (2)
4.All five roomates can take a course in algorithms next year.	(Conjunction?) Modus ponens using (3)

c)

Step	Reason
1.All movies produced by John Sayles are wonderful.	Premise
2.John Sayles produced a movie about coal miners.	Premise
3.A movie about coal miners produced by John Sayles is wonderful.	Universal instantiation from (1), (2)
4.There is a wonderful movie about coal miners	Existential generalization from (3)

d) One version of the answer could be:

Step	Reason
1.There is someone in this class who has been to France.	Premise
2.Every one who goes to France visits the Louvre.	Premise
3.There is someone in this class who has visited Louvre.	Hypothetical syllogism from (1) and (3)

7 Exercise 1.6.28 (15)

Step	Reason
1. $\forall x(P(x) \vee Q(x))$	Given
2. $P(x) \vee Q(x)$	Universal Instantiation from (1)
3. $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$	Given
4. $(\neg P(x) \wedge Q(x)) \rightarrow R(x)$	Universal Instantiation from (3)
5. $\neg R(x) \rightarrow \neg(\neg P(x) \wedge Q(x))$	Contrapositive of (4)
6. $\neg R(x)$	Push Premise
7. $\neg(\neg P(x) \wedge Q(x))$	Modus ponens using (6) and (5)
8. $\neg\neg P(x) \vee \neg Q(x)$	DeMorgan's Law (7)
9. $P(x) \vee \neg Q(x)$	Double Negation (8)
10. $P(x)$	Resolution (2) and (9)
11. $\neg R(x) \rightarrow P(x)$	Pop Premise from (6) to (10)
12. $\forall x(\neg R(x) \rightarrow P(x))$	Universal Generalization (11)

another solution using contrapositive:

Step	Reason
1. $P(x) \vee Q(x)$	Premise
2. $\neg P(x) \wedge Q(x) \rightarrow R(x)$	Premise
3. $\neg P(x)$	Push Premise
4. $Q(x)$	Modus tollens of (1) and (3)
5. $\neg P(x) \wedge Q(x)$	Conjunction of (3) and (4)
6. $R(x)$	Modus ponens of (2) and (5)
7. $\neg P(x) \rightarrow R(x)$	Pop Premise
8. $\neg R(x) \rightarrow P(x)$	Contrapositive

8 Exercise 1.7.18 (12)

Claim: If $n \in \mathbb{Z}$ and $3n + 2$ is even, then n is even.

For now, we will take some basic facts and properties of integers as if they are given.

a) Proof by Contrapositive.

Step	**Reason**
1. n is odd	Push Premise
2. $n+1$ is even	from (1)
3. $\exists k \in \mathbb{Z}(n+1=2k)$	from (2)
4. $n+1=2k$	Existential Instantiation (3)
5. $3n+3=3(n+1)=3(2k)=2(3k)$	Multiply equation in (4) by 3
6. $\exists m \in \mathbb{Z}(3n+3=2m)$	Existential Generalization (5)
7. $3n+3$ is even	Modus ponens using (6) and (5)
8. $3n+2$ is odd	from (7)
9. n is odd $\rightarrow 3n+2$ is odd	Pop Premise from (1) to (8)

b) Proof by Contradiction.

Step	**Reason**
1. n is odd $\wedge\ 3n+2$ is even	Premise for Contradiction
2. $3n+2$ is even	Simplification from (1)
3. $3n$ is even	from (2)
4. n is odd	Simplification from (1)
5. 3 is odd	Basic Fact
6. 3 is odd $\wedge\ n$ is odd	Conjunction of (5) and (4)
7. $(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x$ is odd $\wedge\ y$ is odd $\rightarrow xy$ is odd$)$	Property of $\mathbb{Z}$, use as a Given
8. 3 is odd $\wedge\ n$ is odd $\rightarrow 3n$ is odd	Universal Instatiation (7)
9. $3n$ is odd	Modus Ponens (6) and (8)
10. $3n$ is even $\wedge\ 3n$ is odd	Conjunction of (3) and (9)
11. $\neg(n$ is odd $\wedge\ 3n+2$ is even$)$	(1) leads to contradiction (10)