CSE191, Fall 2013   Assignment 6   Due Fri. 11/1, in class

Reading: As stated before, skim Chapter 3, and especially ignore the proof given about the Halting Problem. The one fact related to Chapter 3 that I will lean on is that if $p(x)$ is a polynomial of degree d with leading term ax^d, and $q(x)$ is a polynomial of degree d with leading term bx^d, then not only are they "Theta-Of" each other, but we get the stronger statement that as x goes to infinity, their ratio $p(x)/q(x)$ converges to a/b. The upshot of this is that when comparing polynomials for their growth rates, only the degree and the leading coefficient matter—all the lower-order terms do *not* matter. An example of this alluded to in lecture is that $\sum_{k=0}^{n} k^r$, which the text gives the exact formula for on page 166 for $k = 1, 2, 3$, is approximately $n^{r+1}/(r+1)$. You can get this approximation by pretending the sum is really an integral—here's where it helps to start the sum from 0 not 1—and doing it. The basic fact is that the integral approximation of a sum is not only "good up to Theta," in fact its ratio to the exact sum approaches 1. That is, it's like the case where a and b are equal, both equal to $1/(r+1)$.

Hence read Chapter 4, sections 4.1–4.3 for next week. These and induction in Chapter 5 will be the other major topics before the second prelim exam, which is likely to be held on Friday Nov. 22.

(1) Rosen, back on page 126, problem 26. (6 pts.)

(2) Rosen, back on page 126, problem 40. Explain why they are not the same. Then explain why they are essentially the same, by giving 1-1 correspondences between both and the set $\{(a, b, c, d) : a \in A \land b \in B \land c \in C \land d \in D\}$. (3+6=9 pts.)

(3) Rosen, page 137, problem 42 done this way: Flip a coin. If the coin comes up heads, do the problem in the text. If the coin comes up tails, instead say whether it follows that $(A \oplus B) \oplus (C \oplus D) = A \oplus ((C \oplus B) \oplus D)$. In both cases, you may assume that the sets A, B, C, D are being represented as discussed in lecture by binary vectors with arithmetic modulo 2—look at question 56 on the same page. (Note that many sources write symmetric difference as $A \triangle B$. If you do both, your score is the average of the square roots of your scores, squared and rounded down. 9 pts.)

(4) Rosen, page 137, problem 50, (c) and (d) only. Show your work, i.e., give reasoning as well to make a "college answer." (6+6 = 12 pts.)

(5) Rosen, page 153, problem 22(b,c). If you say it is not a bijection because it is not a function, can you "patch" it by excluding one or more isolated bad points from the domain? If you say it (or your "patch" of it) is a function but not a bijection, say which of being 1-1 or onto fails, or both. If you say it is not onto, can you "patch" it further by excluding one or more isolated bad points from the range? If your "patch" then becomes a bijection, prove it by showing how given any y in the patched range, you can uniquely solve the equation you get for x. (24 pts. total)

(6) Rosen, page 168, problem 12, (a,b,c) only. For (c), note that you can divide out $(-4)^{n-2}$ from both sides. (3+3+9 = 15 pts.)

(7) Rosen, page 169, problem 32, (b,d) only (9+6 = 15 pts., for 90 on the set).