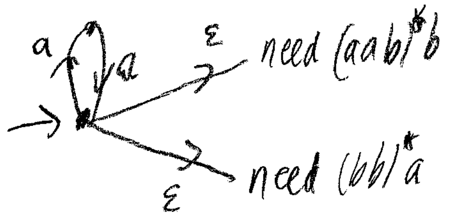
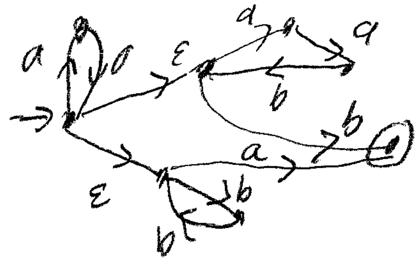
CSE 396 Problem Set 6 Answer Key (Parts (1)–(3)) Spring 2010

(1) NFA for $(aa)^* \cdot ((aab)^* b + (bb)^* a)$: Highest operator is the $\cdot$ after $(aa)^*$, so we focus on that. Next-highest is the lone $+$, so we make a choice with ϵ-arcs.

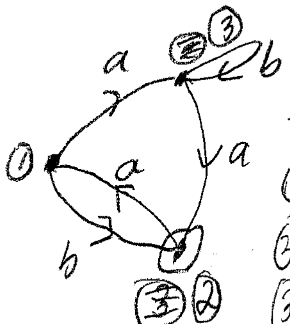


That has broken the problem down enough to complete the NFA in full: N =



(2)

$N_1 =$



After renumbering (2) as (3) and vice-versa:

$T_1 =$

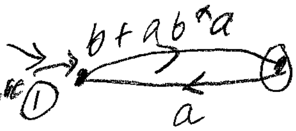
	1	2	3
1	$\emptyset$	b	a
2	a	$\emptyset$	$\emptyset$
3	$\emptyset$	a	b

Eliminate (3): In: $T(1,3)$ $\therefore$ only update $T(1,2)$

Out $T(3,2)$

$T(1,2)_{new} = T(1,2)_{old} + T(1,3)T(3,3)^* T(3,2)$

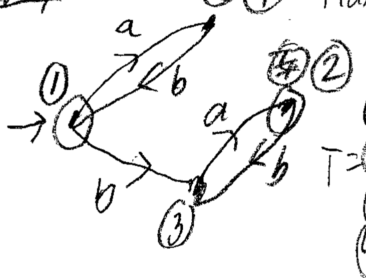
$= b + ab^*a$. Resulting GNFA:



No self-loops, so answer is clear: $(b+ab^*a)\cdot(a(b+ab^*a))^*$.

Sightreading: The two paths from (1) to finish are b and ab^*a. Plussing them together gave the new arc.

$N_2 =$



Re-number by swapping (2) & (4)

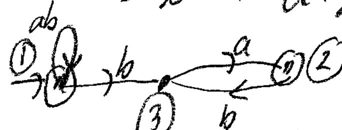
T =

1	$\emptyset$	$\emptyset$	B	a
2	$\emptyset$	$\emptyset$	b	$\emptyset$
3	$\emptyset$	a	$\emptyset$	$\emptyset$
4	b	$\emptyset$	$\emptyset$	$\emptyset$

Elim (4): In $T(1,4)$ out: $T(4,1)$ $\therefore$ do $T(1,1)$

$T(1,1)_{new} = T(1,1)_{old} + T(1,4)\,T(4,4)^*\,T(4,1)$

$= \emptyset + a \cdot \emptyset^* \cdot b = ab$ (recall $\emptyset^* = \epsilon$!)



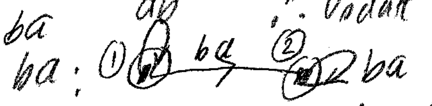
Elim (3): In $T(1,3), T(2,3)$

Out: $T(3,2)$

$\therefore$ Update $T(1,2), T(2,2)$

$T(1,2)_{new} = T(1,2)_{old} + T(1,3)\,T(3,3)^*\,T(3,2) = \emptyset + b\emptyset^* a = ba$

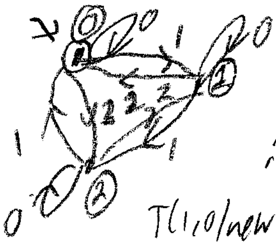
$T(2,2)_{new} = T(2,2)_{old} + T(2,3)\,T(3,3)^*\,T(3,2) = \emptyset + b\emptyset^* a = ba$:



This is the basic 2-state GNFA with both states accepting — and no back-edge from (2) to (1), so it's straightforward: $L_{11} + L_{12}$ where $L_{12} = L_{11}\, ba\, L_{22}$. Since $L_{11} = (ab)^*$, $L_{22} = (ba)^*$, we get $(ab)^* + (ab)^* ba (ba)^*$. This reduces to $(ab)^*(\epsilon + ba(ba)^*) = (ab)^*(ba)^*$. Sightreading: the L_{11}, L_{12} reasoning in the original machine is very similar to that in the reduced one!

(3) $L = \{x \in \{0,1,2\}^* : \sum x_i \equiv 0 \mod 3\}$.

the DFA you should design is clear:



Elim (2) Incoming: $T(0,2), T(1,2)$

Outgoing: $T(2,0), T(2,1)$

$\therefore$ all of $T(0,0), T(0,1), T(1,0), T(1,1)$ need updating

$T(0,0)_{new} = T(0,0)_{old} + T(0,2)\,T(2,2)^*\,T(2,0)$

$= 0 + 20^*1$

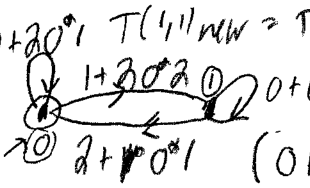
$T(1,0)_{new} = T(1,0)_{old} + T(1,2)\,T(2,2)^*\,T(2,0)$

$= 2 + 10^*1$

$T(0,1)_{new} = T(0,1)_{old} + T(0,2)\,T(2,2)^*\,T(2,1)$

$= 1 + 20^*2$

$T(1,1)_{new} = T(1,1)_{old} + T(1,2)\,T(2,2)^*\,T(2,1) = 0 + 10^*2$



$L_{00} = (T_{00} + T_{01}(T_{11})^* T_{10})^* =$

$(0+20^*1 + (1+20^*2)(0+10^*2)^*(2+10^*1))^*$

Unlike in (2), I don't see how to "sightread" this!