CSE396 **Prelim II** **Nov. 19, 2003**

Closed book, closed notes (except for 1 sheet), closed neighbors, 48 minutes. Do BOTH problems *in the exam booklets provided*. Problem (2) has a *choice*—you must do EXACTLY ONE of the options (a), (b), or (c). Note also an optional alternative in problem (1), part (c). *Show all your work in the exam booklets*—this may help for partial credit. The exam totals 80 pts., subdivided as shown.

Notation: Difference of sets is written $A \setminus B = \{ x : x \in A \wedge x \notin B \}$.

(1) (56 pts. total)

Let $\Sigma = \{ a, b \}$. Let $E = \{ x \in \Sigma^* : |x| \text{ is even and } |x| \geq 2 \}$. Consider the context-free grammar $G = (\{ S, Y \}, \Sigma, \mathcal{R}, S)$ with rules $\mathcal{R}$ given by

$$\begin{aligned} S &\longrightarrow aSa \mid bY \mid Yb \\ Y &\longrightarrow YS \mid a \mid b \; . \end{aligned}$$

(The period is just punctuation.)

(a) Is G ambiguous? If you say yes, find an ambiguous string and give two different parse trees for it. If you say no, prove it. (8 pts.)

(b) Prove by structural induction that $L(G) \subseteq E$. (Reasonable proof shortcuts are OK. 18 pts.)

(c) The grammar's designer intended $L(G) = E$, but there's a "bug." Find three different strings in $E \setminus L(G)$. (Or optionally, write a regular expression r that matches infinitely many strings in $E \setminus L(G)$. 6 pts.)

(d) *Strengthen* the property "P_S" you wrote in part (b) to make a property P'_S that demonstrates "at a glance" that S does not derive any of your strings in part (c). (You need not re-do a structural induction proof with your new P'_S. If the P_S you chose in part (b) already works here, you automatically get full credit for part (d). 6 pts.)

(e) Add one rule to create a context-free grammar G' such that $L(G') = E$. Prove that $E \subseteq L(G')$ by induction on strings. (*Hint:* Separate into cases n even and n odd, and do subcases based on the first and/or last bits of x. 18 pts.)

(2) (24 pts.)

Define $L = \{ a^i b^j c^k : i + j = k, \; i, j, k \geq 0 \}$. Do EXACTLY ONE of options (a), (b), or (c). In whichever case you choose, you must write some prose comments that explain how your G, M_1, or M_2 works—and you need not prove correctness formally.

(a) Design a context-free grammar G such that $L(G) = L$. Then modify your G into a context-free grammar G' such that $L(G') = L \setminus \{ \epsilon \}$ and G' has no ϵ-rules. OR,

(b) Design a single-tape deterministic Turing machine M_1 such that $L(M_1) = L$. OR,

(c) Design a deterministic pushdown automaton M_2, coded as a two-tape deterministic Turing machine that obeys the pushdown restrictions, such that $L(M_2) = L$.

END OF EXAM.