You must turn in the answers to this homework set as hard-copy on 8.5 × 11 in. paper, with your name(s) and user name(s) at the top. Staple multiple pages once in the upper-left hand corner. Write extremely neatly. Anything unreadable will be considered incorrect.

1. (3) Using the Fitch-style proof theory presented in lecture, prove that
\[ \exists x P(x), \forall x \forall y (P(x) \land P(y) \Rightarrow R(x, y)) \vdash \exists x (P(x) \land \forall y (P(y) \Rightarrow R(x, y))) \]

2. (3) Using the Fitch-style proof theory presented in lecture, prove that
\[ \exists (P(x) \land \forall y (P(y) \Rightarrow R(x, y))), \forall x \forall y \forall z (R(x, y) \land R(x, z) \Rightarrow R(y, z)) \vdash \forall x \forall y (P(x) \land P(y) \Rightarrow R(x, y)) \]

3. (10) For each of the following pairs of wffs: if they unify, show an mgu; if they fail to unify, say so and give the reason. Assume that: \( P \) and \( Q \) are predicate symbols; \( f \) and \( g \) are function symbols; \( a, b, \) and \( c \) are individual constants; \( x, y, \) and \( z \) are variables.
   - (a) (2) \( P(a, b, c) \) and \( Q(a, b, c) \)
   - (b) (2) \( P(a, x, c) \) and \( P(a, b, y) \)
   - (c) (2) \( P(a, x, c) \) and \( P(y, b, y) \)
   - (d) (2) \( P(f(a), x, c) \) and \( P(y, g(y), z) \)
   - (e) (2) \( P(f(x), x, c) \) and \( P(y, g(y), z) \)

4. (3) Show the substitution that results from the following substitution composition. Assume that: \( f \) and \( g \) are function symbols; \( a, b, \) and \( c \) are individual constants; \( u, v, w, x, y, \) and \( z \) are variables.
\[ \{ u/x, f(v)/y, w/z \} \circ \{ a/x, b/v, f(u)/w, g(c)/z \} \]

Continued on next page.
5. (3) Translate
\[ \forall x (\exists y P(x, y) \iff \forall y \exists z R(x, y, z)) \]
into clause form. Show all steps. Don’t show any step where nothing changes.

6. (3) Using resolution refutation prove that
\[ \exists x P(x), \forall x \forall y (P(x) \land P(y) \Rightarrow R(x, y)) \vdash \exists x (P(x) \land \forall y (P(y) \Rightarrow R(x, y))) \]