The Jobs Puzzle
A Challenge for Logical Expressibility and Automated Reasoning

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Why I’m here

1984  Wos, Overbeek, Lusk, & Boyle publish the Jobs Puzzle.

ca. 1984–93  I start including the Jobs Puzzle as a standard SNePS demonstration.

Early 2010  I start preparing the Jobs Puzzle as a demo for my KR-2010 poster, and realize that it’s very difficult for resolution reasoners.

Fall 2010  I decide to discuss the situation with the commonsense reasoning community.

There are four people: Roberta, Thelma, Steve, and Pete.

Among them, they hold eight different jobs.

Each holds exactly two jobs.

The jobs are: chef, guard, nurse, telephone operator, police officer (gender not implied), teacher, actor, and boxer.

The job of nurse is held by a male.

The husband of the chef is the telephone operator.

Roberta is not a boxer.

Pete has no education past the ninth grade.

Roberta, the chef, and the police officer went golfing together.

Question: Who holds which jobs?
The Solution by Person or Persons Unknown

<table>
<thead>
<tr>
<th></th>
<th>Roberta</th>
<th>Thelma</th>
<th>Steve</th>
<th>Pete</th>
</tr>
</thead>
<tbody>
<tr>
<td>chef</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>guard</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>nurse</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>operator</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>police</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>teacher</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>actor</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>boxer</td>
<td>no</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

“x” = possibility still in doubt

[based on Wos et al. p 56]
The Solution by Program or Programs Known (OTTER)

Example clauses from

http://www.mcs.anl.gov/~wos/mathproblems/jobs.txt
(accessed 3/2/2011)

POSSJOBS(l(pj(Roberta,chef),
    l(pj(Roberta,guard),
    l(pj(Roberta,nurse),
    l(pj(Roberta,clerk),
    l(pj(Roberta,police),
    l(pj(Roberta,teacher),
    l(pj(Roberta,actor),
    l(pj(Roberta,boxer),
    end)))))))).

-POSSJOBS(l(pj(x,y),l(pj(x,z),end)))
    |  EQUALP(x,w)
    |  EQUAL(pj(w,y),crossed).
“Make no mistake, the representation of the problem to an automated reasoning program is sometimes difficult and sometimes tedious.”
[p.63]
The Challenge

- Represent the Jobs Puzzle to an automated reasoning program, suitable for general-purpose commonsense reasoning,
- in a non-difficult, non-tedious way,
- by a series of logical formulae that adhere closely to the English statements of the puzzle and the allowed immediate inferences,
- and have that automated reasoning program solve the puzzle quickly.
Solutions by:

TPTP Participants (formalized by them)

SNePS (formalized by me)

Lparse/Smodels (formalized by me)
TPTP Overview

- PUZ019-1 in version 5.1.0 of TPTP
  (Thousands of Problems for Theorem Provers)
- 64 Clauses
  - 4 Non-Horn clauses
- Solved by 20 of 29 recorded attempts
- Will show here as standard FOL
SNePS Overview

- SNePS 2.7.1
- SNePSLOG
- Natural deduction
- Sound, not complete
- No modus tollens
Lparse/Smmodels Overview

- Lparse front-end
- Extended logic programming syntax
- Smodels: Stable model semantics
- Finds satisfying models of ground clauses
Unique Names

\[ \forall (x)(equal\_people(x, x) \land equal\_jobs(x, x)) \]
\[ \forall (x, y)(equal\_people(x, y) \Rightarrow equal\_people(y, x)) \]
\[ \forall (x, y)(equal\_jobs(x, y) \Rightarrow equal\_jobs(y, x)) \]

\[ \neg equal\_people(roberta, thelma) \ldots (6\ \text{clauses}) \]
\[ \neg equal\_jobs(chef, guard) \ldots (28\ \text{clauses}) \]

Built in in SNePS

Built in in Lparse/Smmodels
Set/Conjunctive Arguments

SNePS:
\[ P(\{a_1, \ldots, a_n\}) \models P(a_i), \ 1 \leq i \leq n \]

Lparse/Smodels:
\[ P(a_1; \ldots; a_n) \]
abbreviates conjunction of \( P(a_1), \text{ and } \ldots, \text{ and } P(a_n) \)
Counting Propositions & Instances

SNePS:
\[
\text{\texttt{nexists}}(i,j,k)(x)(P(x):Q(x))
\]
k individuals satisfy \(P(x)\),
and, of them,
at least \(i\) and at most \(j\) also satisfy \(Q(x)\)

\[\text{Lparse/Smodels:} \quad i \{ R(x,y)[:P(x)], Q(z) \} j\]
The number of literals that satisfy \(R(x,y)\) plus those that satisfy \(Q(z)\)
(assuming that the first argument of each \(R\) satisfies \(P(x)\))
is between \(i\) and \(j\) inclusive.
1. *jp: There are four people: Roberta, Thelma, Steve, and Pete.*

\[ \forall x(\text{has\_job(roberta, } x) \lor \text{has\_job(thelma, } x) \]
\[ \lor \text{has\_job(pete, } x) \lor \text{has\_job(steve, } x)) \]

\text{Person(\{Roberta, Thelma, Steve, Pete\}).}

\text{person(roberta;thelma;steve;pete).}
inf: “if the four names did not clearly imply the sex of the people, [the puzzle] would be impossible to solve.”

\[ \forall x (\text{male}(x) \lor \text{female}(x)) \land \neg (\text{male}(x) \land \text{female}(x)) \]
\[ : - \text{person}(X), \text{male}(X), \text{female}(X). \]

female(roberta) \land female(thelma)  
male(steve) \land male(pete)

Female({Roberta, Thelma}).  
Male({Steve, Pete}).

female(roberta; thelma).  
male(steve; pete).
2. *jp*: Among [the people], they hold eight different jobs.

4. *jp*: The jobs are: chef, guard, nurse, telephone operator, police officer (gender not implied), teacher, actor, and boxer.

\[ \forall x (\text{has_job}(x, \text{chef}) \lor \text{has_job}(x, \text{guard}) \lor \text{has_job}(x, \text{nurse}) \lor \text{has_job}(x, \text{operator}) \lor \text{has_job}(x, \text{police}) \lor \text{has_job}(x, \text{teacher}) \lor \text{has_job}(x, \text{actor}) \lor \text{has_job}(x, \text{boxer})) \]

3. *jp*: Each holds exactly two jobs.

\[ \forall (x, y, z, u) (\text{has_job}(z, y) \land \text{has_job}(z, x) \land \text{has_job}(z, u) \Rightarrow \text{equal_jobs}(x, y) \lor \text{equal_jobs}(u, y) \lor \text{equal_jobs}(u, x)) \]
2. *jp*: Among [the people], they hold eight different jobs.
3. *jp*: Each holds exactly two jobs.

\[
\text{all}(p) \ (\text{Person}(p) \\
\quad \Rightarrow \ \text{nexists}(2,2,8)\ (j) \ (\text{Job}(j) : \ \text{hasJob}(p,j))). \\
2 \ \{\text{hasJob}(X,Y) : \ \text{job}(Y)\} \ 2 : - \ \text{person}(X).
\]

4. *jp*: The jobs are: chef, guard, nurse, telephone operator, police officer (gender not implied), teacher, actor, and boxer.

\[
\text{Job}(\{\text{chef, guard, nurse, operator, police, teacher, actor, boxer}\}). \\
\text{job}(\text{chef}; \text{guard}; \text{nurse}; \text{operator}; \text{police}; \text{teacher}; \text{actor}; \text{boxer}).
\]
inf: “No job is held by more than one person.”

∀(x, y, z)(has_job(x, z) ∧ has_job(y, z)
          ⇒ equal_people(x, y))

all(j)(Job(j)
         => nexists(1,1,4)(p)(Person(p):hasJob(p,j))).

1 {hasJob(X,Y): person(X)} 1 :- job(Y).
5. *jp*: *The job of nurse is held by a male.*

\[ \forall x (\text{has\_job}(x,\text{nurse}) \Rightarrow \text{male}(x)) \]

all(x) (Female(x) => ~hasJob(x,nurse)).

male(X) :- person(X), hasJob(X,nurse).
6. *jp: The husband of the chef is the telephone operator.*

\[ \forall x (\text{has\_job}(x, \text{chef})) \implies \forall y (\text{husband}(x, y) \iff \text{has\_job}(y, \text{operator})) \]

```prolog
hasJob(X, operator) :- person(X;Y),
                    hasJob(Y, chef),
                    hasHusband(Y, X).
```

```prolog
hasHusband(Y, X) :- person(X;Y),
                   hasJob(Y, chef),
                   hasJob(X, operator).
```

*Not needed for solution*
6. *jp:* The husband of the chef is the telephone operator.
   *inf:* “the implicit fact that husbands are male”
   *inf:* since the chef has a husband, she must be female.

\[
\forall (x, y) (\text{husband}(x, y) \implies \text{female}(x) \land \text{male}(y))
\]

\[
\text{all}(w) (\text{Female}(w) \implies \neg \text{hasJob}(w, \text{operator})).
\]
\[
\text{all}(m) (\text{Male}(m) \implies \neg \text{hasJob}(m, \text{chef})).
\]

2 \{ \text{female}(X), \text{male}(Y) \} 2 :- \text{person}(X;Y),
\]
\[
\text{hasHusband}(X, Y).
\]
7. \textit{jp}: Roberta is not a boxer.

\[ \neg \text{has\_job(roberta, boxer)} \]

\[ \neg \text{hasJob(Roberta, boxer)} . \]

\[ : - \text{hasJob(roberta, boxer)} . \]
8. \textit{jp: Pete has no education past the ninth grade.}

\[ \neg \text{educated}(\text{pete}) \]

\[ \neg \text{educated}(\text{Pete}) . \]

\[ : - \text{ educated}(\text{pete}) . \]
8. inf: “the jobs of nurse, police officer, and teacher each require more than a ninth-grade education.”

\[ \forall x (\text{has\_job}(x, \text{nurse}) \lor \text{has\_job}(x, \text{police}) \lor \text{has\_job}(x, \text{teacher}) \Rightarrow \text{educated}(x)) \]

\[ \text{all}(x) \left( \neg \text{educated}(x) \Rightarrow \text{nor}\{\text{has\_job}(x, \text{nurse}), \text{has\_job}(x, \text{police}), \text{has\_job}(x, \text{teacher})\} \right). \]

\text{educated}(X) :-
  1 \{\text{has\_job}(X,\text{nurse}), \text{has\_job}(X,\text{police}), \text{has\_job}(X,\text{teacher})\} 2, \text{person}(X). \]
9. *jp: Roberta, the chef, and the police officer went golfing together.*

*inf: “Thus, we know that Roberta is neither the chef nor the police officer.”*

\[ \neg (\text{has\_job}(\text{roberta, chef}) \lor \text{has\_job}(\text{roberta, police})) \]

\[ \text{nor}\{\text{hasJob}(\text{Roberta, chef}), \text{hasJob}(\text{Roberta, police})\} \].

\[ 0 \{\text{hasJob}(\text{roberta, chef}), \text{hasJob}(\text{roberta, police})\} 0. \]
inf: “Since they went golfing together, the chef and the police officer are not the same person.”

\[ \forall x \neg (has\_job(x,\ chef) \land has\_job(x,\ police)) \]

\[ all(p) (\text{Person}(p) \Rightarrow \text{nand}\{\text{hasJob}(p,\ chef), \ \text{hasJob}(p,\ police)\}). \]

\[ 0\{\text{hasJob}(X,\ chef), \ \text{hasJob}(X,\ police)\}1 :- \text{person}(X). \]
jp: Question: Who holds which jobs?

∃(x1, x2, x3, x4, x5, x6, x7, x8)(has_job(x1, chef) ∧ has_job(x2, guard) ∧ has_job(x3, nurse) ∧ has_job(x4, operator) ∧ has_job(x5, police) ∧ has_job(x6, teacher) ∧ has_job(x7, actor) ∧ has_job(x8, boxer))

ask hasJob(?p, ?j)?

#hide.
#show hasJob(X,Y).
The answers:

SZS answers short \([\text{thelma, roberta, steve, pete, steve, roberta, pete, thelma}]\)

0.182411 seconds of total run time

(by SNARK)

\[
\text{wff111!}: \quad \text{hasJob(Thelma, boxer)} \\
\text{wff101!}: \quad \text{hasJob(Pete, operator)} \\
\text{wff99!}: \quad \text{hasJob(Pete, actor)} \\
\text{wff87!}: \quad \text{hasJob(Steve, nurse)} \\
\text{wff85!}: \quad \text{hasJob(Roberta, guard)} \\
\text{wff83!}: \quad \text{hasJob(Roberta, teacher)} \\
\text{wff28!}: \quad \text{hasJob(Thelma, chef)} \\
\text{wff24!}: \quad \text{hasJob(Steve, police)}
\]

CPU time : 0.19
Answer: 1
Stable Model: hasJob(pete, operator)
hasJob(pete, actor)
hasJob(steve, nurse)
hasJob(steve, police)
hasJob(thelma, chef)
hasJob(thelma, boxer)
hasJob(roberta, guard)
hasJob(roberta, teacher)

Duration: 0.000
TPTP clause version

- Still somewhat tedious
- Some formalizations more clever than direct translations
- Uses non-Horn clauses
- 9 of 29 recorded attempts failed
- Success required careful choice of strategies
**Conclusions**

**SNePS and Lparse/Smodels versions benefit from**
- Unique Names Assumption
- Set/Conjunctive arguments
- Numerical Quantifier/Cardinality constraints

**SNePS version**
- Natural Deduction and incompleteness provided focus
- Contrapositives occasionally required
- Quite close translation

**Lparse/Smodels version**
- Constraint-satisfaction model-finding
- Limited to ground predicate logic
- Very close translation
Try your favorite system!
Thanks to:

- Mark Stickel
- Raphael Finkel
- SNePS Research Group, especially
  - William J. Rapaport
  - Jonathan P. Bona
  - Michael Kandefer
  - Michael Prentice
  - A. Patrice Seyed
- US Army Research Office (ARO)