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PARAMETRIC TOPOLOGY OPTIMIZATION TOWARD RATIONAL DESIGN & EFFICIENT PREFABRICATION FOR ADDITIVE MANUFACTURING

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ABSTRACT
The significant advance in the boosted fabrication speed and printing resolution of additive technology has considerably increased the capability of achieving product designs with high geometric complexity. The prefabrication computation has been increasingly important and is coming to be the bottleneck in the additive manufacturing process. In this paper, the authors devise an integrated computational framework by synthesizing the parametric level set-based topology optimization method with the DLP-based SLA process for intelligent design and additive manufacturing of not only single material structures but also multi-scale, multi-functional structures. The topology of the design is optimized with a new distance-regularized parametric level set method considering the prefabrication computation. offering the flexibility and robustness of the structural design that the conventional methods could not provide. The output of the framework is a set of mask images which can be directly used in the additive manufacturing process. The proposed approach seamlessly integrates the rational design and manufacturing to reduce the complexity of the computationally-expensive prefabrication process. Two test examples, including a freeform 3D cantilever beam and a multi-scale meta-structure, are utilized to demonstrate the performance of the proposed approach. Both the simulation and experimental results verified that the new rational design could significantly reduce the prefabrication computation cost without affecting the original design intent or sacrificing original functionality.

INTRODUCTION
Additive Manufacturing (AM) or 3D printing has been hailed as the third industrial revolution in the unique way that products are designed and manufactured [1]. Due to the elegant concept of the layer-by-layer fabrication, AM can build complex objects with a wide variety of materials and functions. This opens up tremendous possibilities for complex multi-scale, multi-functional design, and manufacturing with many potential applications, including aerospace, automotive, defense, biomedical and energy industries [2]. Driven by the increasing

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variety of functional materials and the geometric flexibility, AM enables the exploration of new design concepts that would not have been feasible in the traditional paradigm of single-scale, single-material production, due to the limitation of traditional subtractive manufacturing processes [3]. However, the extraordinary design complexity introduced by the expanded design freedom poses a serious challenge towards the prefabrication computation (slicing, path planning, and support generation) under the constraints of limited computational resources [4]. Such challenges usually come from the shape complexity, hierarchical complexity, material complexity and functional complexity of the freeform designs enabled by AM [3].

During the last twenty-five years, the advances in material, process, and machine development have allowed AM processes to evolve from rapid prototyping to rapid manufacturing with the substantially improved manufacturing speed and production throughput [5]. The recent progress in continuous 3D printing technology reduces the manufacturing time by several orders of magnitude [6], which opens up tremendous opportunities for mass customization. Multi-scale metallic metamaterials 3D printing technology has been recently reported, which can fabricate hierarchical metamaterials with different three-dimensional features spanning seven orders of magnitude, from nanometers to centimeters [7]. The significant progress and advancement in the boosted fabrication speed and printing resolution, however, are increasing the complexity of the product design. The prefabrication computation is increasingly expensive and becomes the bottleneck of the additive manufacturing process.

Despite the apparent significance of prefabrication computation in 3D printing, relatively few studies have focused on speeding up prefabrication computation. The current state of the art of prefabrication is delineated in the next section. There is an urgent need to address these challenges, unlock the unique design and manufacturing opportunities enabled by AM, and promote its wide adoption to achieve the full industrial revolution. Our purpose in this paper is to investigate and devise an integrated computational framework for rational design and efficient additive manufacturing of multi-scale, multi-functional structures by seamlessly synthesizing the innovation intelligence from topology optimization and additive manufacturing.

Our rationale is that even though the geometries of multi-scale 3D designs are complicated, these designs have many different configurations but with the same effective physical properties. The complex multi-scale, multi-functional designs provide a substantial rational design space to tailor the geometric configuration for efficient additive manufacturing processing. The computational paradigm of multi-scale, multi-functional AM should not directly repeat the same prefabrication computation. Based on this logic, we put forth a new paradigm of 3D printing for efficient prefabrication computation. This paradigm explores the rational design space of multi-scale, multi-functional through a distance-regularized parametric level set topology optimization to speed up the prefabrication.

The prefabrication bottleneck is largely induced by the continuous 3D printing (CLIP) due to the considerably increased layer numbers, and by the multi-scale printing due to the increased orders of length scale. In both processes, the Digital Light Processing (DLP) projection-based StereoLithogrAphy (SLA) technique is used, and the input of the printing is a set of binary/grayscale images [6, 7]. In this article, the DLP-based SLA process is selected as the typified additive manufacturing process to investigate the prefabrication acceleration by level set based topology optimization method. The same approach can be easily extended to other AM technologies. The proposed parametric level set-based topology optimization approach can speed up the prefabrication process. The parametric level set method provides a "boundary-based" representation of the design to handle shape and topology changes. By the parametric scheme, the level set function is transformed into a weighted summation of kernel function values on selected nodes inside the design domain. Apart from the numerical advantage adopted from the parametric scheme, in computational and in topological aspect, the level set function itself can directly be analytically expressed. In this case, high-resolution image data is readily available for the DLP-based SLA process by directly saving the layer information of the analytical level set function expression, avoiding the computationally expensive slicing and tool path planning procedures. Specifically, the repetitive lattice structures are expected to reduce the prefabrication cost without sacrificing original product functionality.

The remaining paper is arranged as follows: First, the standard flow of the prefabrication and the related challenges are outlined. Second, a level set based topology optimization approach is introduced to explore the design space considering the prefabrication computation cost. Both high-fidelity boundary representation and topology optimization of repetitive lattice structure are proposed to speed up the prefabrication process. Two test cases, including a freeform rational design and a multi-scale design, are utilized to demonstrate the validity of the proposed approach in the following section. A brief discussion with a conclusion is provided at the end of the paper.

PREFABRICATION PROCESS FOR ADDITIVE MANUFACTURING: STATE OF THE ART

Additive Manufacturing (AM) is a class of manufacturing techniques that builds solid objects from computer data directly [3]. The object is fabricated by accumulatively delivering energy and/or material to the designated location. Based on particular energy (heat, light, ultrasound, and so forth) or materials (liquid, solid, powder), AM comprises a collection of various techniques such as Fused Deposition Modeling (FDM), Selective Laser Sintering (SLS), StereoLithogrAphy (SLA). Although the material forming process is different among these techniques, they all start with a CAD model and then transform the digital model into machine instructions, by which the physical object is fabricated with a layer by layer basis.

Standard flow of prefabrication

The regular flow of the 3D model processing includes slicing, tool path planning, and support generation, termed as *prefabrication* in this paper. Before prefabrication, the 3D CAD model will be converted into a polygon mesh in the format of .STL file, which stores a set of triangular faces representing the boundary of the model. Given a model ready for 3D printing, an important preprocess is to convert the model into the data with the prescribed format to guide the operation of 3D printers. The prefabrication of geometric processing of the major 3D printing process includes contour slicing, tool path planning and support generation. Contour slicing is to slice the 3D digital model into a set of thin 2D planar layers; tool path planning is to turn the 2D sliced contours into 1D paths to guide the tools, in order to produce the desired shape; support generation is to create support structures such as a scaffold to hold the overhanging features from dropping to the ground or caving into the liquid materials. However, traditional prefabrication algorithms fall short for the complex multi-scale multi-functional and/or mass customized objects. Specifically, the traditional slicing algorithms generate the contours by visiting highly disordered edges for each layer, either time-consuming sorting algorithm [8, 9] or sophisticated topology reconstruction procedure [10, 11] has to be conducted which severely increases the computational cost. The traditional path planning algorithms generate the tool paths through rasterizing all the contours for each layer [3, 12]. With the improvement of printing resolution and an increase of part complexity, the time complexity is significantly increased accordingly. Similarly, traditional support generation algorithms generate the support structures through examing all the faces of the model and topological connectivities of the features [13-15]. They generally dominate the whole prefabrication process and are sensitive to the complexity of the input model.

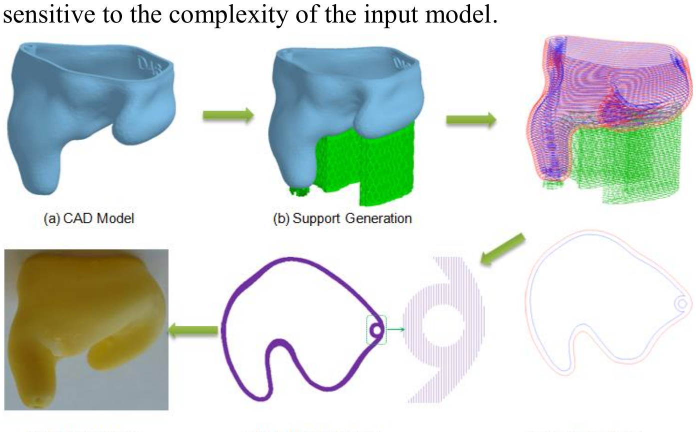


Figure 1. The traditional prefabrication framework for 3d printing

The standard data flow of the AM process is shown in Figure 1. As shown in Figure 1(a), it starts from a digital model. This model has to be processed through support generation, contour slicing, and tool path planning according to its feature. Then the generated tool path is fed into a 3D printer to build a physical object shown in Figure 1(e). In order to ensure sufficient dimensional accuracy and surface smoothness, the size of the triangular facet has to be small enough. That is, a complex model usually requires a significant number of triangular facets which will dramatically increase the input size for the prefabrication. Therefore, the computational cost also increases drastically.

Continuous 3D printing and prefabrication challenges

The emerging Continuous Liquid Interface Production (CLIP) technique, proposed by Tumbleston et al. [6], is a revolutionary breakthrough for AM. Different from traditional AM process which creates objects in a layer-by-layer fashion, CLIP builds objects by continuously depositing energy required for photopolymerization. Its nature of continuity makes it 25-100 times more time-efficient than most SLA based AM machines. On the one hand, the continuous mode allows us to use extremely small layer thickness to represent the delicate local feature change of the model. One the other hand, the adoption of subtle layer thickness or enormous number of layers severely increases the computational cost for prefabrication.

Using the example of the hearing aid shell shown in Figure 1(a) as an example, to offer sufficient dimensional accuracy and surface smoothness, more than one million triangular facets are required to represent the digital model. With CLIP, for a small layer thickness, e.g., 10 μm or even 1 μm, the prefabrication computation needs hours to finish, whereas the printing job on the machine only takes minutes. Therefore, the prefabrication whose computational cost used to be trivial now becomes the major bottleneck of AM. Thus, it is desirable to devise a new fabrication paradigm to significantly improve the entire time efficiency for AM process.

Rational design for prefabrication-favored structure

The expanded design freedom introduces unprecedented design complexity and poses a significant challenge towards the prefabrication computation. However, the expanded design freedom also enables people to reinvent the current computational paradigm and address the prefabrication computation challenges. Since different designs can achieve the same product functionality, we propose a new computational paradigm to take advantage of the high design freedom. The proposed approach converts the design problem into a topology optimization problem with the functionality as the objective. The optimized design is expected to be favorable for prefabrication computation without compromising the original functionality. The comprehensive approaches for prefabrication-favored rational design are explained in the following sections.

PARAMETRIC TOPOLOGY OPTIMIZATION

There is ample rational design space in the multi-scale additive manufacturing. The basic mechanism of this work is to exploit this design freedom and speed up the prefabrication computation process. The reason of the feasibility of the proposed approach lies in that: the nature design (Figure 5a) comes from the evolution with the timeline of millions of years. However, nature evolution ignores the prefabrication constraints. One the other hand, the intuitive inspiration based artificial design relies heavily on designers' intuition and analogy to existing technologies. This is limited by the designers' experiences, and usually, the final solution can only be achieved in a relatively small design space.

The rational design such as the level set-based topology optimization can provide an ultimate solution and has the potential to meet the challenges in prefabrication computation. By utilizing the capability of topology optimization in discovering innovative designs, we hope to discover prefabrication friendly designs without compromising the performance of the design. To accomplish this objective, we employ a distance-regularized parametric level set model with a high-fidelity boundary representation in this study, which enables a direct coupling between topology optimization and additive manufacturing with highly improved prefabrication computation efficiency.

Parametric level set based topology optimization

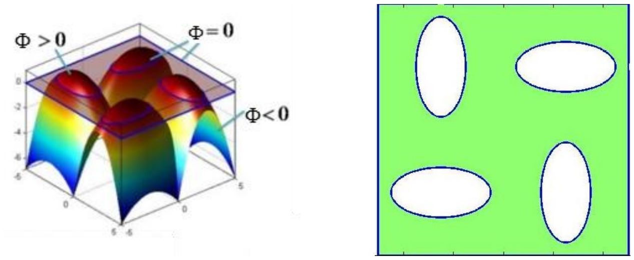
Distance-Regularized Parametric Level Set Model: Traditional density-based topology optimization approaches provide the optimal design with blurred boundaries, so the optimized design cannot be immediately employed to manufacturing processes without post-processing [16]. One the other side, level set methods [17-19] provide a boundary-based clear representation of the topological design and possess unique capabilities of handling free-form geometries (both shape and topology) and topological variations. Level set methods, first proposed by Usher and Sethian [20] were used for tracking moving fluid fronts evolution. Level set methods implicitly represent the design boundaries as the zero level set of a level set function, which has one higher dimension[21], guaranteeing clear design boundaries. During the topology optimization process, the level set function is driven by the design velocity field, calculated from shape sensitivity analysis [22]. To generate high-resolution slice images from the topology optimization result, an analytic level set function is preferred so Bitmap images can be directly interpolated at high accuracy. The analytic formulation of the level set function can be achieved by a Radial Basis Function (RBF) based parametrization scheme [23]. According to [24], the parameterization scheme offers preferable features to the level set-based topology optimization process, including the elimination of artificially imposed shape control of the level set function, no need for design velocity field extension, higher geometry flexibility, and robustness. To overcome some numerical issues in RBF-based parametric level set methods, such as the level set evolution fluctuation, the parametric level set method can be further regularized by introducing a distance-regularization energy functional. By regularizing the level set function to be the desired distance-regularized shape, the numerical fluctuation issues can be eliminated, and extra benefits like higher material interpolation accuracy and topological flexibility can be achieved.

To be more specific, in this paper, a distance-regularized parametric level set method is proposed as an improved computational design approach to deal with the optimal configuration synthesis problem with single or multiple material designs. A distance regularization energy functional [25] is introduced to the level set evolution to regularize the level set function. With this energy functional, the level set function is regularized to be a signed-distance function along the boundary and at the same time remaining flat at other places. This type of the level set function is termed as the distance-regularized level set function. With the distance regularization energy functional, the level set function can perfect the design results with less numeric fluctuation, higher material interpolation accuracy, and more topological flexibility and robustness. The parametric scheme provides the capability of coupling the optimization process with mathematical programming. What's more, the parametric scheme transfers the original PDE driven level set function updating procedure to an ODE driven one, which can ensure a higher computational efficiency and accuracy. The most important for the proposed scheme from the 3D printing point of view is that the parametric level set function can be analytically expressed. This means the level set function can be interpolated and saved as high-resolution slice images, making it directly ready for CLIP 3D printing without any pre-fabrication procedure.

As the name implies, the level set method implicitly represents the boundary as the zero level set of one higher dimensional surface $\Phi(\mathbf{x})$, which is called the level set function. In the level set model, the domain is separated into three parts according to the value of the level set function:

$$\begin{cases} \Phi\left(\mathbf{x}(t)\right) > 0 & : \quad \mathbf{x}(t) \in \Omega \\ \Phi\left(\mathbf{x}(t)\right) = 0 & : \quad \mathbf{x}(t) \in \Gamma \\ \Phi\left(\mathbf{x}(t)\right) < 0 & : \quad \mathbf{x}(t) \in D \setminus \Omega \end{cases}, \tag{1}$$

where D denotes the design domain and $t \in R^{+}$ is time. A sketch of the domain and the corresponding level set representation is shown in Figure 2. The advantage of this implicit boundary representation lies in its ability to naturally handle topological changes, such as splitting and merging of the design boundaries, and at the same provides clear design boundaries directly without any need for post-processing.



(a) A level set function (b) Corresponding boundary

Figure 2. Level set representation of a 2D design.

The above level set function $\Phi(\mathbf{x})$ can be parameterized by using Radial Basis Functions (RBFs). A Level Set Function $\Phi(\mathbf{x})$ can be parameterized into:

$$\Phi(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i \varphi_i\left(\left|\mathbf{x}-\mathbf{x}_i\right|\right), \quad \mathbf{x} \in \mathbf{R}^d, \tag{2}$$

where α_i is a real-valued weight. $|\bullet|$ denotes the Euclidean norm and $\varphi_i(\mathbf{x})$ is the selected RBF $\varphi_i : \mathbf{R}^+ \to \mathbf{R}$. $r = |\mathbf{x}-\mathbf{x}_i|$ is the Euclidian distance between the two points $\mathbf{x}$ and $\mathbf{x}_i$.

With the parametric level set model, the topology optimization problem can be solved with mathematical programming tools, such as CONLIN [26], the Optimality Criteria methods [27, 28] or the Method of Moving Asymptotes (MMA) [29, 30]. In this paper, we use MMA.

Besides the parameterization scheme, a distance-regularization energy functional [25] is introduced to regularize the level set function during the optimization process. A general form of the distance-regularization energy functional can be formulated as

$$R \triangleq \int_{\Omega} P_2\left(\left|\nabla\Phi\right|\right) d\Omega, \tag{3}$$

where

$$P_2 = \begin{cases} \dfrac{1}{(2\pi)^2}\left[1-\cos\left(2\pi\left|\nabla\Phi\right|\right)\right], & \left|\nabla\Phi\right| < 1 \\ \dfrac{1}{2}\left(\left|\nabla\Phi\right|-1\right)^2, & \left|\nabla\Phi\right| > 1 \end{cases} . \tag{4}$$

In equation (4), $P_2\left(\left|\nabla\Phi\right|\right)$ is the distance regularization potential energy, with Φ indicating the level set function and $\left|\nabla\Phi\right|$ representing the norm of $\nabla\Phi$.

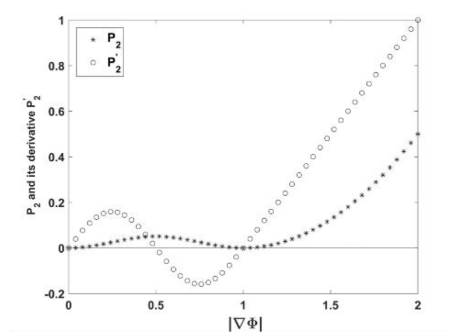
By minimizing the distance regularization energy functional, a level set function with the preferred signed-distance property transition zone along the boundary ($\left|\nabla\Phi\right| = 1$) and a flat surface in the remaining area ($\left|\nabla\Phi\right| = 0$) can be achieved, as shown in Figure 3(c). With the parametric level set method, the optimization problem for a minimum mean compliance structure can be formulated as:

$$\begin{aligned} &\underset{\boldsymbol{\alpha}}{Minimize} : J(u,\alpha) = \int_D j(u) H(\Phi) d\Omega + wR \\ &sub.to: \\ &\int_D H(\Phi) d\Omega - V_{const} \le 0, \\ &a(u,v) = L(v) \quad u|_{\partial D_u} = u_0 \quad \forall v \in U \end{aligned} . \tag{5}$$

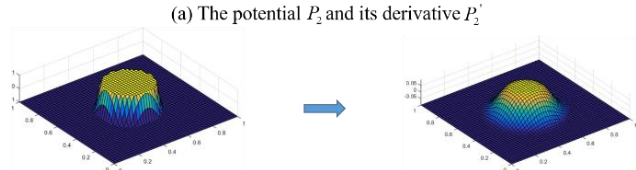
Here $J(u,\Omega)$ is the objective functional and $j(u)$ is the strain energy density. The volume is constrained to be less than or equal to V_0. A distance-regularization energy functional is added to the objective function with a weighting factor w. In equation (5), $a(u,v,\Phi)$ and $L(v,\Phi)$ are the functions for energy and load respectively. They are formulated as follows:

$$\begin{aligned} a(u,v,\Phi) &= \int_D E_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(u) H(\Phi) d\Omega \\ L(v,\Phi) &= \int_D pv H(\Phi) d\Omega + \int_D \tau v \delta(\Phi) \left|\nabla\Phi\right| d\Omega \end{aligned} . \tag{6}$$

In the following section, we will apply the level set based parametric topology optimization approach to efficient prefabrication computation.



(a) The potential P_2 and its derivative P_2'



(b) The Level Set Function before minimizing the potential P_2

(c) The Level Set Function after minimizing the potential P_2

Figure 3. (a)The double-well potential P_2 and its derivative P_2' (b) The initial step function for distance-regularization (c) The final distance-regularized level set function

PARAMETRIC TOPOLOGY OPTIMIZATION FOR EFFICIENT PREFABRICATION COMPUTING

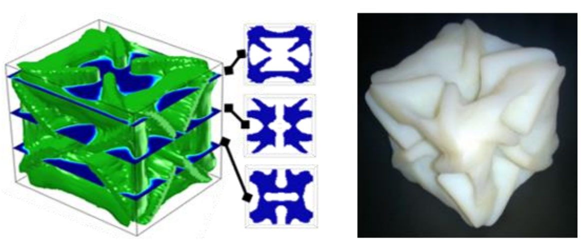
High-fidelity boundary representation for high-efficiency prefabrication computation

The traditional geometric representation for 3D printing is based on triangle mesh surfaces, which pose significant challenges to the data representation and process for hierarchical structures. In our proposed approach, because the analytic level set function is known, the boundary of the design can be calculated with high precision, which will be useful in the multiscale 3D printing of small features. Due to the continuous parametric representation, the geometric features can be very small up to the 3D printer's physical resolution. The proposed representation is invariant to the complexity of the hierarchical structure.

Also, with the parametric RBF level set model, the prefabrication (i.e., slicing and path planning) of the geometric model becomes easy to implement. When the level set based topology optimization process finishes, the analytic level set function can be identified with coefficients α_i and corresponding radial basis functions ϕ_i, as shown in Equation (2). By introducing a set of grid $\mathbf{x}_n$ determined by the pixel resolutions of the binary image for 3D printing, the analytic level set function can be interpolated to $\mathbf{x}_n$ with arbitrary accuracy following equation (7)

$$\Phi(\mathbf{x}_n) = \sum_{i=1}^{n} \alpha_i \phi_i \left(\left| \mathbf{x}_n - \mathbf{x}_i \right| \right). \quad (7)$$

Here, the desired structural boundaries, interpolated as the zero contour (in 2D) or the iso-surface (in 3D) of the level set function, can be analytically calculated. Since the zero contour (in 2D) or the slices of the zero contour (in 3D) of the level set function can be directly saved as Bitmap (.bmp) images , the slice layer information can be directly generated and saved.



(a) (b)

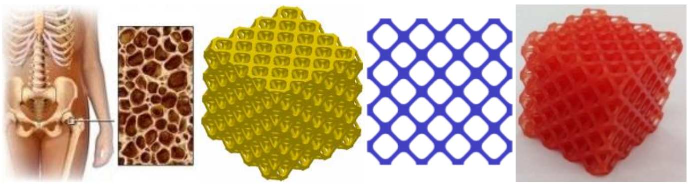
Figure 4. (a) A 3D negative Poisson's ratio structure [31] and its layer images. (b) High-speed 3D DLP-SLA printing

With the saved high-resolution layer images, the geometric model can be directly integrated with high-resolution and high-speed 3D DLP-SLA printing [6], as shown in Figure 4. Suppose the physical resolution of a 3D printer is $m \bullet n \bullet p$, and the number of triangles is *k*, where *m* and *n* are the numbers of pixels in horizontal direction, and *p* is the number of layers in the vertical direction. Originally, the slicing and tool path planning algorithms must sweep over each triangle and calculate the intersection points between the cutting plane and the edges of the triangles, and then rasterize the image based on the intersection line segments, making the aggregate time complexity $m \bullet n \bullet p \bullet k$. To represent the small features in the hierarchical structure, a significant number of triangles are required, which immediately increased the time complexity of the prefabrication computation. In our parametric RBF level set based model, the level set function can easily judge the material status of each voxel, that is, for the voxel *x*, $\Phi(x) < 0$ indicates the material should be deposited at position *x*. Otherwise, no material should be printed. Therefore, the time complexity is measured as $m \bullet n \bullet p$, which depends only on the resolution of the printer. This new representation can also be easily extended to multi-material, multi-functional structure. By formulating a parametric model for each material and incorporating multi-physics law in the topology optimization stage, the multi-material and multi-functional design can be conveniently integrated with the proposed scheme.

Topology optimization of repetitive lattice structures

Nature enables the optimal designs with complex geometries. The emerging additive manufacturing technology allows for the conversion from compound designs to real 3D objects. However, lacking efficient prefabrication computational tool leads to the gap between the design and manufacturing. To bridge this gap, we propose a topology optimization based repetitive internal lattice structure for non-periodic biologic ultra-structures. It is expected that the proposed design will be favorable for the prefabrication computation without compromising the design object performance. Specifically, the computational cost will be reduced by orders of magnitude compared with the original complex design.

An illustrative example is shown in Figure 5. The nature design (Figure 5a) of the bone tissue indicates an extremely complex structure with random internal porous structure. This complicated porous structure requires huge memory storage (measured in Gigabytes or Terabytes) and computational cost (measured in hours to days). Our proposed topology optimization based approach considers the physical performance of the structure and redesigns it as repetitive lattice configuration in Figure 5(b). The new rational design can achieve the same or similar functionality compared to the nature design. What's more, due to the repetitive configuration, only the simple atomic element, and the repetitive rules are needed to be stored and processed, indicating a dramatically reduced memory usage (measured in Kilobytes or Megabytes) and computation cost (measured in seconds or minutes). Figure 5(c) shows a typical layer path planning by copying the element and propagating it in 2-dimensional direction; the same procedure will also be performed in the third dimension. Only small numbers of elements are repeated in this test case for visualization purpose; the practical design could include millions of such elements. The repetitive porous structure is not only used in biological applications but also wider industrial areas, for example, those lightweight structures are highly preferred in aerospace applications.



(a) (b) (c) (d)

Figure 5. (a) Nature design;(b) Repetitive design; (c) Mask image for a typical layer of the printing; (d) 3D printed part

EXPERIMENTAL RESULTS

Two test problems including a freeform cantilever beam and a multi-scale metamaterial are utilized to verify the performance of the proposed approach.

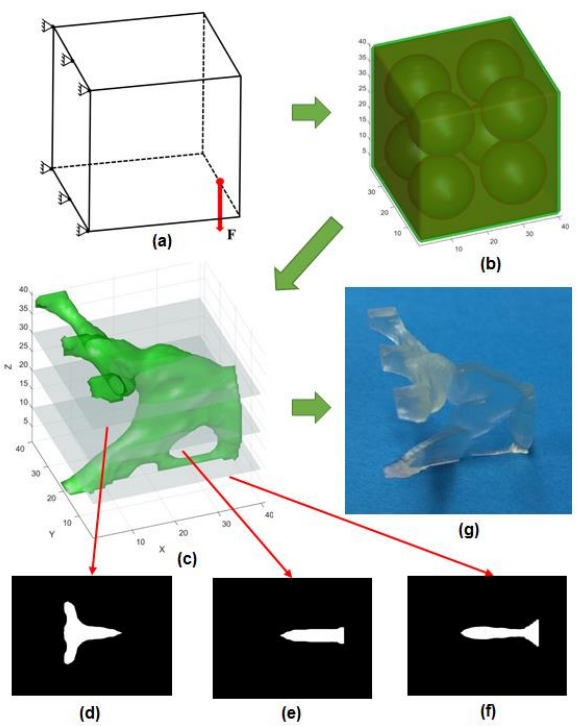


Figure 6. The generation of the parametric level set based rational design of a freeform structure. (a) The design boundary conditions; (b) The initial design; (c) The final design; (d) The optimized design model from the proposed approach; (e-f) Mask images for three patterned layers at different height; (g) The 3D printed part based on the image set through a continuous printing process.

Figure 6 shows the optimized design of the 3D cantilever beam, following the same optimization setting in equation (5). The intent of demonstrating the freeform cantilever beam is to verify the effectiveness of smooth integration of the design and prefabrication so the prefabrication can be trivially implemented by directly converting the design into input data for the printing process. Based on the optimal design and the corresponding level set function, high-resolution layer images can be generated to an arbitrary accuracy by simply evaluating the parametric function at the specific pixel position of the image. Figure 6(c) shows the optimized 3D cantilever beam design. Figure 6(d-f) show three mask images of patterned layers at different heights. Figure 6(g) shows the printed physical part with the set of mask images through a continuous 3D printing process. The efficiency of the prefabrication process will be discussed at the end of this section.

With the proposed distance-regularized parametric level set scheme, the structural topology optimization can be extended from single, uniform material structure to many complex problems such as multi-scale or multi-functional structures. The parametric level set provides the freedom of the selecting of design variables, and the distance-regularization effect ensures an accurate material property interpolation.

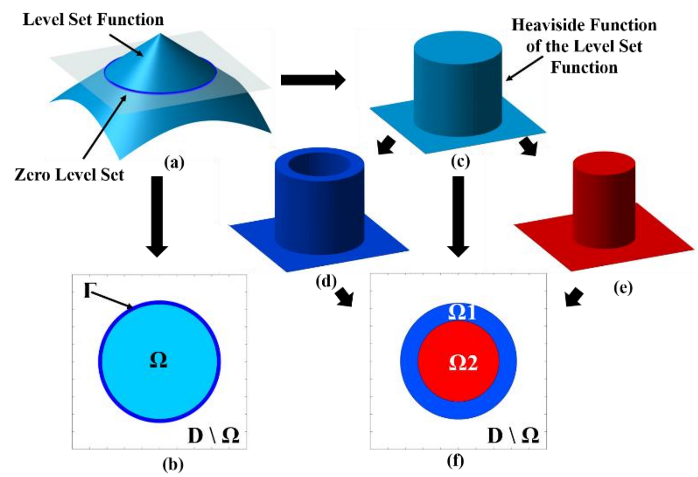


Figure 7. Multi-material property interpolation from the level set function. (a) Level set representation; (b) The zero level set of the level set function. The whole design domain is divided into the material domain Ω, material boundary Γ, and the void domain D\Ω; (c) The Heaviside function of the given level set function; (d) The Heaviside function for the 1st material property; (e) The Heaviside function for the 2nd material property; (f) The final Heaviside function for the multi-material representation.

To separate different material properties in the proposed distance-regularized multi-material parametric level set method, the mapping from the level set function to structural material property needs to be specified according to the value of the level set function. As shown in Figure 7(a), (b) and (c), when the value of the level set function is above zero, the Heaviside function of the level set function equals to one, and it is zero for the remaining design domain. Here, the material region Ω and the void region D\Ω are interpolated via the value of the Heaviside function value. To interpolate multiple material properties in the material region, the original Heaviside function of the level set function is separated into two parts with a specific bandwidth based on the design needs (as shown in Figure 7(d) and (e)). Different material properties are interpolated to the material region based on the distance information, which is illustrated in Figure 7(f). For instance, given the material property P to be interpolated in the material region, with given material property $P1$ in Ω_1, material property $P2$ in Ω_2 and material property P_{void} in the void region, the overall material property P can be expressed as

$$P = HS_1(P1) + HS_2(P2) + HS_{void}(P_{void}), \tag{7}$$

where different HS represents the corresponding Heaviside segment of the level set function.

The repetitive structure comprises millions or billions of the same atomic elements. Therefore, the prefabrication is only required for one element, and the result can be trivially piled in the 3-dimensional space. In our proposed paradigm both the overall structure, shape and the topology of the microstructure can be optimized simultaneously, which realize topology optimization of hierarchical structures.

In the following example, the objective of the design of the unit element is to achieve a negative Poisson's ratio effect for the overall structure. This target can be hit by minimizing the difference between the elastic stiffness constant of the structure design and its target value in the least square manner. Generally, the 2D optimization problem can be formulated as

$$\begin{aligned}
&\underset{\alpha}{Minimize}: J = \frac{1}{2}\sum_{i,j,k,l=1}^{2} w_{ijkl}\left(C_{ijkl}^{H} - C_{ijkl}^{t}\right)^2 \\
&sub.to: \\
&\int_D H(\Phi) d\Omega - V_{const} \le 0, \\
&a(u,v) = L(v) \quad u|_{\partial D_u} = u_0 \quad \forall v \in U
\end{aligned} \tag{8}$$

Here, C_{ijkl}^{H} is the homogenized elastic stiffness constant and C_{ijkl}^{t} is its target with the corresponding weight w_{ijkl}. The boundary conditions are the same as the ones stated in equation (6).

The result of topology optimization of a multi-material hierarchical structure is shown in Figure 8. The hierarchical structure can effectively mimic the nature design such as human bones. As shown in the above figure, the dense walls (blue color) can mimic the compact layer of the bone, which can provide strength and toughness, while the internal cellular fillers (green color) can mimic the spongy layer of the bone, which can absorb shock energy. With the new paradigm improving the prefabrication computation efficiency by several orders of magnitude, the original functionality such as the light weight, stiffness, and bioactivity can be properly preserved. As for this multi-scale design, the internal filler (green color) has a resolution greater than the wall (blue color). However, with the analytic parametric level set function expression, high-resolution layer images can be directly generated to an arbitrary accuracy. In this way, the prefabrication computation cost can be reduced by orders of magnitude. Figure 8(a~d) show the optimized design processing with multi-material representation. Figure 8(e-f) illustrate the input images for both the high-resolution and low-resolution structures. Figure 8(g-h) demonstrate the 3D printed parts, generated from a continuously DLP-based SLA process. The physically fabricated part proves that the proposed approach can increase the prefabrication efficiency without sacrificing the functionality or physical property.

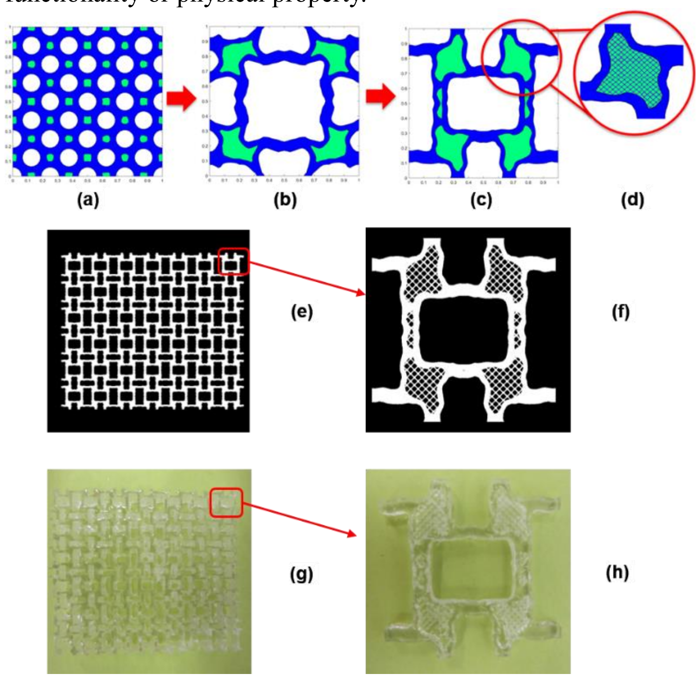


Figure 8. Level set based topology optimization of a hierarchical structure and its 3D printing. (a) Initial design; (b) Intermediate design during the optimization process; (c) Final design with multi-scale structures; (d) The close-up view of the high-resolution internal cellular fillers; (e) The mask image of one typical layer of the structure; (f) The close-up view of the high-resolution mask images; (g) The 3D printed multi-scale part; (h) The close-up view of the local fine structure of the internal fillers.

In the previous section, the theoretical time complexity was analyzed. To further verify the efficiency of the proposed approach in prefabrication process, computational time cost is also evaluated for the two test cases by comparing the proposed rational design based approach and conventional sequential flow based approach. The test platform is a PC with an Intel E5 3.6GHz CPU, and 8Gbytes RAM, running on the Windows 7 operating system. The 3D cantilever beam comprises 650 thousand (650K) faces, and the multiscale model contains 10 million (10M) faces. The layer thickness is set to 1μm to ensure enough resolution for the continuous printing process.

Table 1. The computational time comparison

Model	Size	#Layers	T_t	T_p
Freeform	650K	14K	2h32m	2m40s
Multi-scale	10M	2K	5h41m	1m52s

The time measurement units are in hours (h), minutes (m) and seconds (s). The size of the model is described as the number of faces. #Layers represents the number of the sliced layers. T_t and T_p are the computational time from the conventional approach and proposed approach, respectively. Note that the computational time for topology optimization is not counted toward the comparison between the two approaches.

Table 1 summarizes the computational time statistics of the two test cases. It can be seen that the proposed approach has significantly reduced the prefabrication cost. For the freeform test case, the proposed approach can instantly generate the images by evaluating the parametric level set function in equation (7), while the traditional approach will scan all the triangular faces, construct the contours and restores the contour into images. For the multiscale part, the proposed approach shows even higher computational efficiency, primarily attributed to the repetitive design of the atomic patterns.

CONCLUSION

Level set methods provide a boundary-based representation of the topological design to handle freeform geometries and topological variations. The distance-regularized level set function ensures an accurate and smooth mapping of structural material throughout a stable optimization evolution. Different material properties can be identified via the selection of different sections of the Heaviside function from the level set function, creating a final optimal design with multi-material phases. With the RBF based parametric scheme, the level set function can be analytically interpolated to binary images at high resolution. Therefore the high-resolution image data is readily available from the level set function, bypassing the traditional computationally expensive slicing and path planning procedures. Specifically, the 3D cantilever beam structure and the repetitive lattice structures are demonstrated via the proposed method to show its ability to reduce the prefabrication cost without losing original product functionality. Both the simulation and experimental results provide the verification of this advantage by the numerical comparison.

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