As a TM computes, changes occur to state, tape contents, current head position. A setting of this information is called a configuration.

A snapshot, instantaneous description (≡ D), or configuration of a TM M is a word \( q_1 q_2 \ldots q_n \), where \( q \in Q \) is the current state, \( a_1 a_2 \ldots \) is the contents of the tape (up to the rightmost nonblank or, up to the symbol to the left of the head, whichever is rightmost), and the tape head is scanning the first symbol of \( a_2 \) (or \( B \), in case \( a_2 = \) \( B \)).

Now we describe the next move or yield relation; we denote this with a turnstile \( \vdash \).

Let \( a_1 a_2 \ldots a_i q a_i \ldots a_n \) be a configuration.

Suppose \( \delta(q, a_i) = (p, b, L) \).

If \( i < n \), then

\( a_1 a_2 \ldots a_i q a_i \ldots a_n \vdash a_1 a_2 \ldots \hat{a}_i \ldots a_n \),

\( a_1 \ldots a_i \hat{a}_i p a_1 b a_2 \ldots \hat{a}_i a_n \).
If \( i = 1 \), then

\[ qa_1 \ldots a_n \not\vdash p ba_2 \ldots a_n \]

Suppose \( \delta(q, a_i) = (p, b, R) \)

If \( i - 1 \neq n \) (i.e., \( a_1 \ldots a_n \not= \lambda \)),

then

\[ b \quad c \quad p \quad c \quad a_i \vdash a_i \ldots a_{i-1} bp a_i \ldots a_n \]

If \( i - 1 = n \) (so, \( a_i = B \)),

\[ a_i \ldots a_n \not\vdash a_i \ldots a_n b p \]

The start configuration of \( M \) on \( w \)

is \( q_0 w \)

A configuration is accepting if the state of the configuration is \( \text{accept} \)

\[
\begin{array}{c}
\text{accepting} \\
\text{rejecting} \\
\text{reject} \\
\end{array}
\]

Accepting and rejecting configurations are the only halting configurations.
A TM $M$ accepts input $w$ if

$q_0 \xrightarrow{w} q^* q_1 \xrightarrow{\text{accept}} q_2.$

$C \xrightarrow{*} D$ means that $C = D$ or
there is a sequence of configurations $C_1, ..., C_k$ s.t.

$C = C_1 \xrightarrow{*} C_2 \xrightarrow{*} \cdots \xrightarrow{*} C_k = D$

$M$ rejects $w$ if

$q_0 \xrightarrow{w} q^* q_1 \xrightarrow{\text{reject}} q_2.$

$L(M) = \{ w \in \Sigma^* \mid M \text{ accepts } w \}$

Def. A language $L$ is Turing acceptable if $L = L(M)$ for some Turing machine $M$.

(Do not use recognizable for this!)

Def. A language $L$ is Turing decidable if $L = L(M)$ for some Turing machine $M$ that halts on every input.
Note the distinction!

acceptable:
\[ x \in L \Rightarrow M \text{ eventually enters } \text{ accept} \]
\[ x \not\in L \Rightarrow M \text{ eventually enters } \text{ reject} \]

or
\[ M \text{ runs forever} \]

decidable:
\[ x \in L \Rightarrow M \text{ eventually enters } \text{ accept} \]
\[ x \not\in L \Rightarrow M \text{ eventually enters } \text{ reject} \]
Turing machines can compute functions:

A TM $M$ computes a function $f : (\Sigma^*)^n \rightarrow \Sigma^*$

Place the input $(w_1, \ldots, w_n)$ on the otherwise blank tape so that the initial configuration is $q_0, w_1 B, w_2 B, \ldots, B, w_n$.

(2) $M$ eventually enters an accepting configuration $f(w_1, \ldots, w_n) \in \text{accept}$.

i.e., the tape is empty except for the value of $f(w_1, \ldots, w_n)$, and $M$ halts with the head behind this value.

Homework:
Design a Turing machine to compute and $n^2$ in binary notation. Give an implementation description. (see p. 156) on a multi-tape machine.
Multitape Turing machines:

A k-tape TM (k ≥ 1) has tapes, each with its own read/write head.

\[ \delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k \]

\[ \delta(q, a_1, \ldots, a_k) = (p, b_1, \ldots, b_k, s_1, \ldots, s_k) \]
Ex. A 2-tape TM to recognize \( \{ww^R | w \in \{0,1\}^*\} \).

Copy the input onto the second tape, compare the word claimed to be \( w^R \) on the input tape with the reverse of \( w \) on tape two. Move the heads in opposite directions, and check the length of the input to make sure it is even.

Note that the number of steps is \( O(n) \), \( n = |w| \). A one tape TM requires \( O(n^2) \) steps.

Similarly for \( \{wcwc^R | w \in \{0,1\}^*\} \).