CS531 HW 3 Solution

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1. In the star shaped architecture, there is no way you can communicate with each other without going through PE0. If each PE only holds one entry, the time to do the semigroup operation is

\[(n - 1) + (n - 1) + (n - 1)\]

The first \(n - 1\) is for the communication from PE\(_1\) to PE\(_0\), the 2nd \(n - 1\) is for the computation of semigroup and the last communication is for the communication fr0 PE\(_0\) to PE\(_i\), which is asymptotically identical to the serial computer algorithm. If we want to make the architecture useful, the only way I can think of is to put a great amount data \(k \gg n\) into the architecture.

2. (a) The PEs are grouped into two groups A and B as follows

\[
\begin{array}{ccc}
0 & 1 & 01 \\
2 & 3 & 23 \\
4 & 5 & 45 \\
6 & 7 & 67
\end{array}
\]

The semigroup operation can be done in \(\lg n\) time.

We can exchange the info and calculate the following fashion

\[
\begin{array}{cccccccc}
0 & 1 & 01 & 0123 & 0167 & 0-7 & 0-7 \\
2 & 3 & 23 & 0123 & 2345 & 0-7 & 0-7 \\
4 & 5 & 45 & 2345 & 0167 & 2345 & 0-7 & 0-7 \\
6 & 7 & 67 & 4567 & 0167 & 4567 & 0-7 & 0-7 \\
\end{array}
\]

The algorithm can be written as the following: the variable mid is the id of myself, cid is the id of the PE which the PE communicates in this phase, \(n\) is the number of PEs.

```c
pcid = mid;
for(i = 0; i < \lg n; i++) {
    if(mid \% 2 == 0) cid = (pcid + pow(2, i)) \% n;
    else cid = (pcid - pow(2, i)) \% n;
    exchange info the cid : we have dm (my data) and dr (remote data)
    dm = dm + dr /* Semigroup operation here */
    pcid = cid;
}
```

1
(b) Assume that after we remove the edges from the PEs, the PEs will be split into two sets: set 1 and set 2. If there are \( x \) of PEs in the set A belong to set 1, then in set B, there must be \( n/2 - x \) of PEs belong to set 1 since the total number of the PEs in set 1 are \( n/2 \). Similarly, the number of PEs in set A which belong to set 2 will be \( n/2 - x \) and \( x \) in set B.

\[
\begin{array}{c}
\forall & \forall \\
\uparrow & \uparrow \\
\downarrow & \downarrow \\
\text{set 1} & \text{n/2 - x} & \text{set 1} \\
\downarrow & \downarrow \\
\forall & \forall \\
\uparrow & \uparrow \\
\downarrow & \downarrow \\
\text{set 2} & \text{n/2-x} & \text{set 2} \\
\downarrow & \downarrow \\
\forall & \forall \\
\end{array}
\]

Set A | Set B

The the edge need to be cut off in order to make two disjoint set will be

\[
f(x) = x^2 + (n/2 - x)^2 = 2x^2 - nx + n^2/4
\]

The minimum occurs at the point of \( f'(x) = 0 \), which is

\[
f'(x) = 4x - n = 0, x = n/4
\]

If \( n \) is divisible by 4, then \( x = n/4 \), however, if \( n \) is not divisible by 4, \( x = \lceil n/4 \rceil \) or \( \lfloor n/4 \rfloor \).

Then replace \( x \) back to \( f(x) \), we will get the bisection width of \( 2 \cdot (n/4)^2 - n \cdot n/4 + n^2/4 = n^2/16 \).

The maximum number of the edges in any graph are \( n \cdot (n - 1) = \Theta(n^2) \) which is asymptotically identical to the bisection width. Therefore the architecture is pretty much connected.