First Name (Print): __________________________ Last Name (Print): __________________________

UB ID number: __________________________

1. This is a closed book, and closed neighbor exam. You may use a calculator, and a two-sided sheet of notes.

2. Support your answer.

3. Write your name on the top right-hand corner of every page.

4. There are 6 problems and 25 points in this exam.

5. **Once the instructor announces “time’s up”, you must stop writing immediately. It’s your responsibility to give your exam to TA within 2 mins.**
1 (2 + 2 = 4 points).
   (a) Let $A$ and $B$ be two sets. Suppose that:
      
      - $A - B = \{1, 5, 7, 8\}$;
      - $B - A = \{3, 6, 9\}$;
      - $A \cap B = \{2, 10\}$.

      What is $A$ and $B$?

      $A = (A - B) \cup (A \cap B) = \{1, 2, 5, 7, 8, 10\}$;
      $B = (B - A) \cup (A \cap B) = \{2, 3, 6, 9, 10\}$;

   (b) Let $A, B, C$ be three sets. Suppose $A \cup C = B \cup C$. Can you conclude $A = B$? If your answer is “yes”, prove it. If your answer is “no”, give a counter example.

      No.
      Counter example:
      $A = \{1\}$, $B = \{1, 2\}$, $C = \{1, 2, 3\}$;
      $A \cup C = B \cup C$, but $A \neq B$.

2 (3 points). Let $A$ be the set of binary strings that satisfy the following conditions:

   - The length of the binary string is 7.
   - Either the first two bits are 00; or the last bit is 1.

   Determine $|A|$. (Namely, determine the number of elements in $A$.)

   Let $B$ be the set of 7-bit binary strings of which the first two bits are 00;
   Let $C$ be the set of 7-bit binary strings of which the last bit is 1.

   Then $|B| = 2^5 = 32$, $|C| = 2^6 = 64$.
   Note that $B \cap C$ is the set of 7-bit strings whose first two bits are 00 and last bit is 1.
   So $|B \cap C| = 2^4 = 16$.
   So $|A| = |B \cup C| = |B| + |C| - |B \cap C| = 80$.  

Name

**Note:** For problem 3, the solutions **may** involve factorial, combinations, permutations and powers, such as $6!$, $C(6, 3)$, $P(10, 5)$, or $(-5)^k$. In such cases, your solutions must be given in terms of these expressions. The numerical solution is not enough, and is not required. For example, if the solution is $P(7, 3) \times 2^5$, you can just write it this way, or $7 \times 6 \times 5 \times 2^5$. Its numerical value 6720 is neither enough, nor required.

3 (6 points). How many ways can a photographer at a wedding arrange 7 people (including the bride and the groom) in a row, if:

(a) the bride is NOT next to the groom?

The total number of possible arrangements: $P(7, 7)$.
The total number of arrangements that the bride is at the left and next to the groom: $P(6, 6)$.
The total number of arrangements that the bride is at the right and next to the groom: $P(6, 6)$.
So the answer is: $P(7, 7) - 2 \times P(6, 6)$

(b) the bride is next to the groom?

The total number of arrangements that the bride is at the left and next to the groom: $P(6, 6)$.
The total number of arrangements that the bride is at the right and next to the groom: $P(6, 6)$.
So the answer is: $2 \times P(6, 6)$

(c) the bride is positioned somewhere to the left of the groom?
(Hint: a row satisfies this requirement if and only if the reverse of the row does NOT satisfy this requirement).

Solution 1: There are $P(7, 7)$ ways to arranged 7 people. Exactly half of these arrangements satisfy the requirement. So the answer is $P(7, 7)/2$.

Solution 2: Choose 2 positions for bride and groom, $C(7, 2)$ ways.
Arrange bride and groom in these 2 positions, 1 way (bride on the left side of the groom).
Arrange other 4 people, $P(5, 5)$ ways.
So, there are $C(7, 2) \times 1 \times P(5, 5) = P(7, 7) \cdot \frac{1}{2}$ ways.
Let \( R \) be the set of real numbers; \( R^+ \) be the set of positive real numbers; \( Z \) be the set of integers; \( Z^+ \) be the set of positive integers.

(a) Consider the following functions:

- \( f : R \to R : f(x) = 3x - 1. \)
- \( g : Z \to Z : g(x) = 3x - 1. \)
- \( h : R^+ \to R : h(x) = x^2 + 1. \)

Fill the following table by “yes” or “no”:

<table>
<thead>
<tr>
<th></th>
<th>1-to-1?</th>
<th>onto?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( g )</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>( h )</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

(b) Let \( f, g \) and \( h \) be the functions represented by the following arrow diagrams.

Fill the following table by “yes” or “no”:

<table>
<thead>
<tr>
<th></th>
<th>function?</th>
<th>1-to-1?</th>
<th>onto?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>no</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( g )</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( h )</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Note: because \( f \) is not a function, you don’t need to answer the 2nd and 3rd columns of the 1st row.
5 (2 points) Determine the cardinality of the following sets. (Namely: is it finite? countable infinite? uncountable?)

- \((0, 0.02) = \{x \in R \mid 0 < x < 0.02\}\) = the set of real numbers between 0 and 0.02
  uncountable infinite
- \(P(Z^+)\) = the power set of positive integers. (Namely the set of subsets of positive integers.)
  uncountable infinite
- \(Z^+ \times Z^+ = \{(a, b) \mid a \text{ and } b \text{ are positive integers}\}\)
  countable infinite
- The number of atoms in the solar system.
  finite

6 (2+1+1 = 4 points).

(a) Find the value of the following sum:
\[
\sum_{i=0}^{6} (4^i - 4 \cdot 2^i)
\]
You must use the summation formula, (not by calculating the sum term by term.)
\[
\sum_{i=0}^{6} (4^i - 4 \cdot 2^i) = \sum_{i=0}^{6} (4^i) - \sum_{i=0}^{6} (4 \cdot 2^i) = \frac{4^7 - 1}{4 - 1} - 4 \cdot \frac{2^7 - 1}{2 - 1} = 4953
\]

(b) Find the value of the following sum: \(\sum_{i=0}^{\infty} (3/4)^i = 1 + (3/4)^1 + (3/4)^2 + (3/4)^3 + \cdots\)
\[
\sum_{i=0}^{\infty} (3/4)^i = \lim_{n \to \infty} \frac{(3/4)^n - 1}{3/4 - 1} = 4
\]

(c) Convert the periodic decimal \(x = 0.913\overline{28}\) to a fraction.

Solution 1 (using the formula discussed in class):
\[
x = 0.913\overline{28} = \frac{91328 - 91}{99900}
\]

Solution 2:
\[
x = 0.913\overline{28} = 0.91 + 0.00328(1 + (1/1000)^2 + (1/1000)^3 + \cdots)
\]
\[
= \frac{91}{100} + 0.00328 \cdot \lim_{n \to \infty} \frac{(1/1000)^n - 1}{1/1000 - 1} = \frac{91}{100} + \frac{328}{99900} = \frac{91237}{99900}
\]