Embedding of Meshes on Rotator Graphs[1]

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Abstract – A set of directed permutation graphs called rotator graphs were proposed as an alternative to the star and pancake graphs for multiprocessor interconnection networks. The rotator graphs have a smaller diameter than star and pancake graphs for the same number of nodes, while sharing the properties of star, pancake, and binary hypercubes like maximal fault tolerance, partitionability, etc. In this paper we develop a class of algorithms for recognizing undirected mesh structures in n-rotator graphs. The average dilation of the embeddings are very low as compared to the dilation of the embedding. These embeddings will be very useful for regular computations with bi-directional requirements, in addition to the irregular computations in n-rotator graphs. Most of the results presented here equally apply to another set of directed Cayley graphs, the cycle prefix digraphs.

Introduction

In recent years the symmetric Cayley graphs as possible interconnection networks have been studied by many researchers [1, 2, 3, 4, 5]. Many Cayley graphs have been introduced in recent years [1, 5, 6, 7]. These Cayley graphs are maximally fault-tolerant and have simple routing algorithms [2, 8, 9, 6]. Most of the proposed Cayley interconnection networks are undirected graphs. However, few directed Cayley interconnection networks have also been proposed recently due to their practical importance. Some of the advantages of the directed interconnections are their ease to construct and faster communication [10, 11, 12]. The comparative performance analysis of the directed Manhattan Street Network (MSN) and bi-directional mesh connected (2D-Torus) network by Chung and Agrawal [13] shows similarities in performance of MSN and mesh, while the MSN is cost-effective. Due to the practical importance of the directed communication links, the uni-directional versions of several well studied undirected interconnection networks like the Star graph [14], the Hypercube [12], and meshes [15] have also been proposed and analyzed.

Real world applications present a variety of communication requirements like regular communication patterns (sorting, searching), irregular communication patterns (computer vision, artificial intelligence), all-to-all communication (direct n-body computation, matrix-vector multiplication) [16], one way communication patterns (one-way cellular automata-OCA) [11], etc. Directed graph models have been used for many scientific and engineering applications like neural networks [17]. On the other hand, the computational requirements of most of the parallel algorithms for regular applications often lead to the exchange of information between adjacent processing elements and communication between neighboring processors. Also, many existing algorithms were designed with the assumption of the availability of a shortest bi-directional link between the processing elements. A general purpose interconnection network should efficiently emulate both regular and irregular communication patterns. Embedding algorithms for the star interconnection network have been studied by many authors [18, 19, 20].

The n-rotator graph as an alternative to star and pancake graphs was introduced by Corbett [2]. The rotator graphs are a class of directed interconnection networks. The n-rotator graph has n! nodes and each vertex is represented by a unique permutation of n symbols. The generators of the n-rotator graphs are of the form $(a_1, a_2, a_3, ..., a_i, a_{i+1}, ..., a_k) \rightarrow (a_2, a_3, ..., a_i, a_1, a_{i+1}, ..., a_k)$ where $1 < i \leq k$.

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The cycle prefix symmetric networks introduced by Faber, Moore, and Chen [21, 7] were similarly defined. A simple and optimal routing algorithm for n-rotator graphs is given by Corbett [2]. The diameter and average diameter of the n-rotator graph are lower than that of star and pancake graphs. The star graph and the rotator graph have degree of $(n-1)$, but the average diameter of n-rotator converges rapidly as compared to the star graph. A comparison of the diameter and the average diameter of star, pancake, hypercube, and n-rotator is given by Corbett [10]. The hierarchical and cyclic structure of the n-rotator graph can be used in applications like fast Fourier transforms [22], sorting [23], load balancing, broadcasting, personalized communication [24, 16], etc.

Unlike the star graph which has only even cycles, the n-rotator graph has both odd and even cycles [25]. However, not all generators are immediately reversible in n-rotator graphs. In an attempt to generalize the proposed n-rotator graphs and to solve the conflicting requirements of the nature of computations and the implementation constraints we propose embeddings of undirected networks on directed permutation graphs. We develop a class of algorithms for embedding meshes on n-rotator graphs.

The rest of the paper is organized as follows. In section two we present the definitions to be used in the paper. Section three gives the embedding algorithms for mapping two dimensional meshes on n-rotator graphs. Section four concludes the paper with directives of future research.

Definitions

Definition 1 *A set of interconnected processors with directed edges is denoted as R, and a set of processors with undirected edges is denoted as G. The vertices and edges in G are denoted by $V(G)$ and $E_{x,y}(G)$ respectively, where $E_{x,y}(G)$ is the undirected edge between x and y. The vertices and edges of R are denoted by $\vec{V}(R)$ and $\{\vec{E}_{x,y}(R), \vec{E}_{y,x}(R)\}$ respectively, where $\vec{E}_{x,y}(R)$ denotes the directed edge from the vertex x to the vertex y.*

Definition 2 *The directed path from vertex x to vertex y is denoted by $\vec{E}^P_{x,y}(R)$, where the path P may contain one or more than one directed edges, and an undirected path form vertex x to y can be denoted by $E^P_{x,y}(R)$.*

Definition 3 *The n-rotator graph has n! vertices and each vertex is represented by a unique permutation of n distinct symbols $1, 2, 3, ...n$. The generators are of the form $(a_1, a_2, a_3, ..., a_i, a_{i+1}, ..., a_k) \rightarrow (a_2, a_3, ..., a_i, a_1, a_{i+1}, ..., a_k)$ where $1 < i \leq k$. An n-rotator graph has n disjoint (n -1)-rotator graphs,* i.e., *the (n -1)-rotator graphs are the subgraphs of the n-rotator graph.*

Figure 1 illustrates the 4-rotator graph. It consists of four 3-rotators. The undirected edges are denoted by bold lines and the directed links are denoted by lines with arrows. The undirected edges are due to the reversible generators. In Fig. 1, the 24 nodes are represented by permutations of four symbols 1, 2, 3, and 4. The alphabets are used to denote the directed links which connect the subrotators. Each one of the 24 nodes is associated with an alphabet in addition to the permutations. For example, the alphabet associated with the vertex 1234 is a, where $a(o)$ denotes the directed link which

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originates *from* the node represented by a, i.e., 1234. The symbol $j(i)$ denotes that the input node *to* the vertex 1234 is *from* the vertex which is represented by the alphabet j, i.e., 4123.

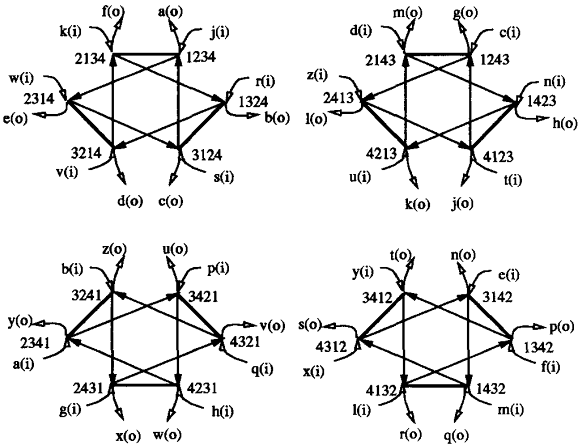

Figure 1: *4-rotator Graph*

Definition 4 *The action of the generators in the n-rotator graph is a rotation of length ℓ, where $2 \leq \ell \leq n$. The permutation $a_2a_3a_4......a_\ell a_1$ $a_{\ell+1}....a_n$ is obtained from the node $a_1a_2a_3.......a_\ell a_{\ell+1}.....a_n$ after applying a rotation of length ℓ or equivalently, the generator g_{l-1}.*

In Fig. 1, the node 4123 is obtained after a rotation of length 4 (g_3) applied to the node 3412. Similarly, a rotation of length 3 applied to node 4123 would result in 1243. Clearly all the generators are not reversible, i.e., a rotation of length four applied to 4123 would not result in 3412, though a rotation of length four applied to 3412 would result in 4123.

Definition 5 *An embedding e of an undirected graph G in a directed graph R is the mapping of the vertices $V(G)$ into the vertices $\vec{V}(R)$ and of edges $E_{x,y}(G)$ into the directed path $\vec{E}^P_{e(x),e(y)}(R)$. An embedding always exists when R is connected and $|R| \geq |G|$. The embedding is said to have a dilation j, where*

$$j = \max_{\forall (x,y)\in \vec{E}(R)} \left|\vec{E}^P_{e(x),e(y)}\right|$$

Definition 6 *Two nodes x and y are said to be at bi-distance l, where*

$$l = \max_{x\neq y}(\vec{E}^P_{x,y}, \vec{E}^P_{y,x}).$$

For example, the bi-distance between the nodes 4321 and 3241 in Fig. 1 is two. Here $\left|\vec{E}^P_{4321,3241}\right| = 1$ and $\left|\vec{E}^P_{3241,4321}\right| = 2$. The bi-distance between the nodes 4321 and 3421 is one as $\left|\vec{E}^P_{4321,3421}\right| = \left|\vec{E}^P_{3421,4321}\right| = 1$. Similarly, the bi-distance between the nodes 4321 and 2314 is three as $\left|\vec{E}^P_{4321,2314}\right| = 2$ and $\left|\vec{E}^P_{2314,4321}\right| = 3$.

Definition 7 *A directed graph is said to have a bi-diameter d_b, if the maximum shortest bi-distance between any pair of vertices in the graph is d_b.*

Definition 8 *An n-rotator is represented by the permutations of 1,2,. ....n symbols. The n-rotator has n $(n-1)$-rotators, and each (n - 1)-rotator has (n - 1)! nodes of permutations with a symbol k = 1,2,........,n fixed at the last position. The (n - 1)-rotator with a fixed k is called the k-fixed-rotator. It is obvious that all $(n-1)$ k-fixed-rotators for $k = 1$ to n, are isomorphic to the $(n-1)$-rotator.*

In Fig. 1, all the permutations end with the symbol 4 in the last position, i.e., 1234, 2134, 2314, 3214, 3124, and 1324 form the 4-fixed-rotator. Similarly, all the nodes with 3 in the last position form 3-fixed-rotator, etc.

Corollary 1 *The bi-distance between two nodes $A = a_1a_2........a_ka_{k+1}$ $........a_n$ and $B = b_1b_2........b_ka_{k+1}........a_n$, is at most k - 1, where $a_1a_2........a_k$ and b_1b_2 $.......a_k$ are the permutations of a group of k numbers, where $0 < k < n$ and the tails of the permutations i.e. $a_{k+1}, ...a_n$, where $0 < n-k < n$, are the same in both the permutations A and B.*

Proof: The tails of the permutations A and B are the same. Thus, by definition, A and B are the vertices of a k subrotator. It is known that the diameter of an n-rotator is $(n-1)$. Therefore, the diameter of the k-subrotator is $k-1$. The k-subrotator is isomorphic to the n-rotator.

Theorem 1 *The minimum bi-distance from any node of a (n-1) subrotator to a node from the other (n-1), (n-1) subrotators of an n-rotator graph is $\lceil n/2 \rceil$.*

Proof: The n-rotator graph with the permutation $a_1a_2........a_n$ has n subrotator graphs of size $n-1$, each of them have (n - 1)! nodes with fixed $a_n = k$, where $k = 1, 2, ..., n$. The nodes with a fixed a_n, *i.e.*, the nodes within an (n-1) subrotator can have bi-distances from 1 to (n-2). However, the minimum number of rotations required to reach a node with a different fixed (a_k) and come back to the original node is exactly n. For example, if a rotation of length n is applied initially, to come back to the original vertex it requires $(n-1)$ rotations, which makes the bi-distance $(n-1)$. However, to find the optimal bi-distance between the nodes of two different subrotators we first apply $n/2$ rotations if n is even, or $\lceil n/2 \rceil$ or $\lfloor n/2 \rfloor$ rotations if n is odd. Obviously we need $n - n/2$ ($= n/2$) or $n - \lceil n/2 \rceil$ ($= \lfloor n/2 \rfloor$) or $n - \lfloor n/2 \rfloor$ ($= \lceil n/2 \rceil$) rotations to come back to the original vertex. This makes the shortest bi-distance between any node of an (n-1) subrotator and a set of nodes of other subrotators to be $\lceil n/2 \rceil$.

Embedding Algorithms

Theorem 2 *The optimal embedding of a mesh in an n-rotator has a dilation $\lceil n/2 \rceil$.*

Proof: This theorem is proved by showing that none of the nodes outside an (n-1) subrotator can be reached in $\lceil n/2 \rceil$ bi-distance [Theorem 1]. For a $j-$*dilation* embedding to exist, at least one node in any of the $(n-1)$ $(n-1)$-subrotators should be at a bi-distance within j from any node of a (n-1)-subrotator. By theorem 1, the minimum bi-distance between any nodes of an $(n-1)$-subrotator and the rest of the n-rotator graph is $\lceil n/2 \rceil$. Therefore, the optimal embedding of a mesh or any other undirected graph on an n-rotator will have a dilation of $\lceil n/2 \rceil$.

Corollary 2 *It is not possible to get a constant dilation embedding of meshs in n-rotator graphs.*

Proof: By constant dilation embedding, we mean that an embedding with dilation k, where k is independent of the size of the n-rotator n. The proof of this corollary follows the proof of theorems 1 and 2. It has been shown clearly that the dilation of the embeddings is dependent on n.

Corollary 3 *There is no dilation 1 embedding of a mesh in an n-rotator graph for $n > 2$.*

Proof: This can be proved by showing that none of the nodes in an n-rotator graph have more than one node which are at bi-distance 1 for $n \geq 3$. Only the 2-rotator graph has a neighbor at bi-distance 1 as the generator is reversible.

$\lceil n/2 \rceil$ *Dilation Embeddings*

Since the lowest possible dilation embedding of a mesh on an n-rotator is $\lceil n/2 \rceil$, we give algorithms for dilation $\lceil n/2 \rceil$ embeddings. The concept of the embedding is that in n-rotator graphs some nodes can be arranged as parallel paths with a ladder like structure. The two columns of the ladder belongs to two different subrotators and the bi-directional distance between the nodes of the ladder is $\lceil n/2 \rceil$. It is possible to maintain the condition that the $\lceil n/2 \rceil$ embeddings resulting in nodes which need $\lceil n/2 \rceil$ steps in one direction and very few steps i.e., one in the other direction, i.e., the average dilation of the embeddings are low as compared to the dilation of the embedding. The higher dilation is due to the directed nature of the n-rotator and the action of the generators.

Theorem 3 *Two dimensional meshes of size* $2 \times \frac{n!}{2}$ *can be embedded in an n-rotator with expansion 1 and optimal dilation* $\frac{n}{2}$ *for even values of n.*

Proof: It is clear from theorem 1 that every node from a $(\frac{n}{2}+1)$-subrotator will have exactly one node at bi-distance $\frac{n}{2}$ in other $\frac{n}{2}+1$ subrotators of the same n-rotator graph. Therefore, there are $(\frac{n}{2}+1)!$ nodes, which could be reached at bi-distance $\frac{n}{2}$ from this subrotator. It should be noted that exactly two nodes out of $(\frac{n}{2}+1)!$ nodes belong to the same $(\frac{n}{2}+1)$-subrotator. In an $(\frac{n}{2}+1)$-subrotator the last $\frac{n}{2}-1$ symbols are already sorted. Therefore, the same symbols should also be in sorted order in the source nodes, i.e., in the source node the symbols $a_{(\frac{n}{2}+2)} \ldots a_n$ and $a_2 \ldots a_{\frac{n}{2}}$ should be in sorted order. Then, the two symbols left are a_1 and $a_{(\frac{n}{2}+1)}$, which could only form two different combinations. By embedding all the nodes of various $(\frac{n}{2}+1)$-subrotators, it is possible to achieve dilation $\frac{n}{2}$.

Table I shows the embedding of a 2×12 mesh on the 4-rotator graph with dilation 2.

TABLE I
DILATION 2 EMBEDDING OF 2×12 MESH IN AN 4-ROTATOR GRAPH

mesh	*rotator*	*mesh*	*rotator*
(1,1)	3124	(7,1)	1342
(2,1)	1234	(8,1)	3412
(3,1)	2314	(9,1)	4132
(1,2)	2134	(7,2)	4312
(2,2)	3214	(8,2)	1432
(3,2)	1324	(9,2)	4123
(4,1)	1423	(10,1)	3241
(5,1)	2143	(11,1)	4321
(6,1)	4213	(12,1)	2431
(4,2)	2413	(10,2)	4231
(5,2)	4123	(11,2)	2341
(6,2)	1243	(12,2)	3421

Theorem 4 *A mesh of size* $l \times m$ *can be embedded in an* n*-rotator with optimal dilation* $\lceil n/2 \rceil$*, where* $l \leq 3(n-1)$ *and* $m \geq \left\lceil \frac{n \times (n-2)!}{3} \right\rceil$*, for odd values of n.*

Proof: It is known that the minimum bi-distance between a set of nodes of a $(\lceil n/2 \rceil+1)$-subrotator and a set of nodes of another $(\lceil n/2 \rceil+1)$-subrotator is $\lceil n/2 \rceil$. It can be observed from the definitions and the directed nature of the n-rotator graph that the mesh structure can be realized only by identifying the parallel set of nodes between subrotators of different sizes. Therefore, in order to find a mapping of a two dimensional mesh on an n-rotator, it is required to find the number of nodes at bi-distance $\lceil n/2 \rceil$ from a set of nodes of one subrotator. Once this set of nodes are found, it is required to find the group of nodes from this set, which are at bi-distance $\lceil n/2 \rceil$ and belong to

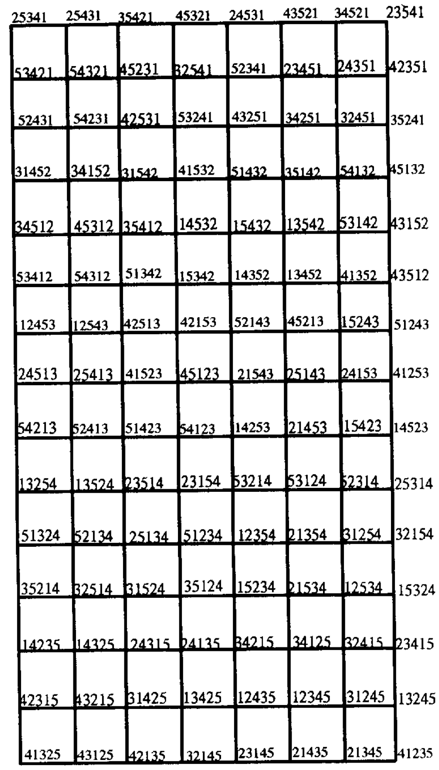

Figure 2: *Dilation 3 embedding of an* 8×15 *mesh in 5-rotator*

another $\lceil n/2 \rceil$ subrotator. It is obvious that all the nodes within the $\lceil n/2 \rceil$ subrotator satisfy the condition, i.e., the nodes can be arranged in two rows of nodes with $\frac{(\lceil n/2 \rceil)!}{2}$ nodes each (with dilation $\lceil n/2 \rceil$). However, it is not possible to obtain another set of $\frac{(\lceil n/2 \rceil)!}{2}$ nodes from the rest of the subrotators which are at bi-distance $\lceil n/2 \rceil$ from these nodes and among themselves.

Consider a vertex represented by $a_1 a_2 a_3 \ldots a_{\lceil n/2 \rceil} a_{\lceil n/2 \rceil+1} \ldots a_n$. According to our definitions, all the nodes within bi-distance $\lceil n/2 \rceil + 1$, also belong to the n-fixed-rotator. It is required to identify the set of nodes satisfying the above conditions. We define a ladder as two sets of nodes from two different $(\lceil n/2 \rceil + 1)$-subrotators that can be arranged like a ladder with $\lceil n/2 \rceil$ bi-distance. The ladders are of the type $p_1 q_1 p_2 q_2 \ldots q_x$, where $q_i p_{i+1}$ form the $ladder(i+1)$ and $\{q_i \cap p_{i+1}\} = 0$. Here the symbols p and q represent different sets of nodes belong to different subrotators. The maximum possible length of these ladders is one important factor in deciding the size of the mesh that could be embedded on an n-rotator. Since the last $\lfloor n/2 \rfloor - 1$ symbols of all the nodes in any $\lceil n/2 \rceil + 1$ subrotator are the same, and they form one side of the ladder the nodes in the next side of the ladder must have these $\lfloor n/2 \rfloor - 1$ symbols from the previous subrotator in the same *order*, in addition to the $\lfloor n/2 \rfloor - 1$ last symbols of the next subrotator. Therefore, there are only three symbols left after subtracting $2 \times (\lfloor n/2 \rfloor - 1)$ from n for odd values of n. These three symbols along with the $\lfloor n/2 \rfloor$ symbols can form $6\lfloor n/2 \rfloor$ possible combinations. Since $6\lfloor n/2 \rfloor$ is the same as $3(n-1)$ for odd values of n the size of the mesh which could be embedded with this dilation is $3(n-1) \times \frac{n!}{3(n-1)}$.

The value of $x = \frac{n!}{\lceil n/2 \rceil + 1}$ is the number of distinct $(\lceil n/2 \rceil + 1)$-subrotators. The subrotators can be arranged in the order of $\binom{n!}{\lfloor n/2 \rfloor - 1}$ non-repetitive permutations first and so on. All the permutations between a p side of the ladder and the adjacent q side of the ladder belong to the same ($\lceil n/2 \rceil + 1$)-subrotator and hence the bi-distance between any of the nodes is always $\lceil n/2 \rceil$, which is the same as $\frac{n-1}{2}$ for odd values of n. The selection of the adjacent ($\lceil n/2 \rceil + 1$)-subrotator is also important for this embedding. It is required to arrange the subrotators so that the last $\lfloor n/2 \rfloor - 1$ symbols in two adjacent subrotators are distinct.

The $ladder(i)$ as defined above can be arranged at positions $0, k, 2k$, $3k.....xk$, for a mapping of an $l \times m$ mesh on an n-rotator, where

$$k \geq \frac{\left(\frac{n+3}{2}\right)!}{3(n-1)}$$

, $xk = m$, $x = \frac{n!}{k \times l}$, and $n!$ should be a multiple of k.

A mapping of 8×15 mesh on a 5-rotator with dilation three using the above algorithm is given in Fig. 2.

The following embeddings are trivial. A 2-dilation embedding of a mesh on an n-rotator is trivial because the value of $\lceil n/2 \rceil$ is the same as the bi-diameter $n-1$. Same is the case in $\lceil n/2 \rceil + 1$ (4) dilation embedding of a mesh on a 4 and 5-rotators.

Similar algorithms can be used for even values of n with dilation $n/2+1$, and for odd values of n with higher dilations for various related networks. The average dilation of the embeddings can be kept very low.

Conclusion

In this paper the embedding characteristics of the recently proposed directed n-rotator graph have been analyzed. Optimal embedding algorithms for both odd and even values of n are given. Since the dilation of the embedding is dependent on the parameter n, the dilation increases with the value of n. However, the work preserving properties and the communication cost of these embeddings remain to be investigated. The extension of this algorithm for multi-dimensional meshes and higher expansion embeddings can also be investigated. Future work include the extension of this embedding techniques to multidimensional meshes and evaluating the complexity of mesh algorithms like meshsort, etc.

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