Written Assignment #1

Content Covered: Summations

Submission Process, Late Policy and Grading

Due Date: 2/4/24 @ 11:59PM
Total points: 50

Your written solution may be either handwritten and scanned, or typeset. Either way, you must produce a PDF that is legible and displays reasonably on a typical PDF reader. This PDF should be submitted via autolab as WA1. You should view your submission after you upload it to make sure that it is not corrupted or malformed. Submissions that are rotated, upside down, or that do not load will not receive credit. Illegible submissions may also lose credit depending on what can be read. Ensure that your final submission contains all pages.

You are responsible for making sure your submission went through successfully.

Written submissions may be turned in up to one day late for a 50% penalty.
No grace day usage is allowed.

Problem 1 - Summations

[50/50 points]

Simplify each of the following equations, $f_i(n)$. Your final result should be a sum-free equation in terms of n. You may use any of the rules discussed in-class (see the summary below). Show your work as a sequence of steps. For each step, indicate the specific rule that relates the equation to the previous one. All logarithms are base-2.

$$f_1(n) = \sum_{i=n}^{2n} \frac{i}{5}$$

$$f_2(n) = \sum_{i=1}^{4} (i^2 + 5)$$

$$f_3(n) = \sum_{i=0}^{n} \frac{\log(2^i \cdot 2^{2^i})}{2}$$

$$f_4(n) = \sum_{i=1}^{n} \sum_{j=0}^{i} 2^j$$

$$f_5(n) = \sum_{i=n}^{n^3} 2i$$

$$f_6(n) = \sum_{i=1}^{2n} \sum_{j=1}^{3n} i$$

$$f_7(n) = \sum_{i=n}^{n^2} 2^i$$

$$f_8(n) = \sum_{i=0}^{10} \sum_{j=1}^{i} n$$

$$f_9(n) = \sum_{i=1}^{log(n)} 3i$$

$$f_{10}(n) = \sum_{i=1}^{n} \sum_{j=1}^{\log(i)-1} 2^j$$

Reference Material

Closed form summation equivalences

The following works for any functions f, g (even constants). c is any constant relative to $i, j, k, \ell \in \mathbb{Z}$. Any sum $sum_{i=j}^{k} f(i)$ is always 0 if $k < j$.

S1. $\sum_{i=j}^{k} c = (k - j + 1)c$

S2. $\sum_{i=j}^{k} (cf(i)) = c \sum_{i=j}^{k} f(i)$

S3. $\sum_{i=j}^{k} (f(i) + g(i)) = \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$

S4. $\sum_{i=j}^{k} (f(i)) = \left(\sum_{i=\ell}^{k} (f(i))\right) - \left(\sum_{i=\ell}^{j-1} (f(i))\right)$‘ (for any ‘$\ell < j$)

S5. $\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \ldots + f(k-1) + f(k)$

S6. $\sum_{i=j}^{k} f(i) = f(j) + \ldots + f(\ell - 1) + \left(\sum_{i=\ell}^{k} f(i)\right)$ (for any $j < \ell \leq k$)

S7. $\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{\ell} f(i)\right) + f(\ell + 1) + \ldots + f(k)$ (for any $j \leq \ell < k$)

S8. $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

S9. $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Closed form logarithm equivalences

L1. $\log(n^a) = a \log(n)$

L2. $\log(an) = \log(a) + \log(n)$

L3. $\log(\frac{n}{a}) = \log(n) - \log(a)$

L4. $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$

L5. $\log(2^n) = 2^{\log(n)} = n$

Example

The derivation to find the closed form for $\sum_{i=0}^{n-2} \sum_{j=0}^{i} 20$ is as follows:

apply **S1** with $j = 0, k = i, c = 20$

$$\sum_{i=0}^{n-2} \sum_{j=0}^{i} 20 = \sum_{i=0}^{n-2} (i - 0 + 1)20 = \sum_{i=0}^{n-2} (i + 1)20$$

apply **S6** with $j = 0, \ell = 1, k = n - 2$

$$= 1 \cdot 20 + \sum_{i=1}^{n-2} (i + 1)20 = 20 + \sum_{i=1}^{n-2} (i + 1)20$$

apply **S2** with $c = 20, f(i) = (i + 1), j = 1, k = n - 2$

$$= 20 + 20 \sum_{i=1}^{n-2} (i + 1)$$

apply **S3** with $f(i) = i, g(i) = 1, j = 1, k = n - 2$

$$= 20 + 20 \left(\sum_{i=1}^{n-2} i + \sum_{i=1}^{n-2} 1 \right)$$

apply **S8** with $k = n - 2$

$$= 20 + 20 \left(\frac{(n-2)(n-1)}{2} + \sum_{i=1}^{n-2} 1 \right)$$

apply **S1** with $c = 1, j = 1, k = n - 2$

$$= 20 + 20 \left(\frac{(n-2)(n-1)}{2} + (n - 2 - 1 + 1) \cdot 1 \right)$$

$$= 20 + 20 \left(\frac{n^2 - 3n + 2}{2} + (n - 2) \right)$$

$$= 20 + (10n^2 - 30n + 20) + (20n - 40)$$

$$= 10n^2 - 10n$$