0 Overview

Instructions

Due Date: Sunday, Feb 25 @ 11:59PM

Total points: 50

Your written solution may be either handwritten and scanned, or typeset. Either way, you must produce a PDF that is legible and displays reasonably on a typical PDF reader. This PDF should be submitted via autolab as WA2. You should view your submission after you upload it to make sure that it is not corrupted or malformed. Submissions that are rotated, upside down, or that do not load will not receive credit. Illegible submissions may also lose credit depending on what can be read. Ensure that your final submission contains all pages.

You are responsible for making sure your submission went through successfully.

Written submissions may be turned in up to one day late for a 50% penalty.

No grace day usage is allowed.

1 Questions

Part 1 - Sorted Lists and Hints

Each of the following questions ask about runtimes related to the `SortedList` class you defined in PA1. When answering, give the most specific runtime bound you can: if a Big-Theta bound exists, give that, otherwise give the tightest upper bound (Big-O) you can. All bounds should be in terms of n. In at most two sentences, explain the runtime you gave. **Answers given without an explanation will receive no credit**.

1. [4 points] What is the runtime of `SortedList.insert(...)`?

2. [4 points] What is the runtime of the following snippet of code?

```
SortedList<Integer> list = new SortedList<>();
for (int i = 0; i < n; i++) {
  list.insert(i);
}
```

3. [4 points] What is the runtime of the following snippet of code?

```
SortedList<Integer> list = new SortedList<>();
LinkedListNode<Integer> hint = list.insert(0);
for (int i = 1; i < n; i++) {
  hint = list.insert(i, hint);
}
```

4. [4 points] What is the runtime of the following snippet of code?

```
SortedList<Integer> list = new SortedList<>();
for (int i = n; i >= 0; i--) {
  list.insert(i);
}
```

5. [4 points] What is the runtime of the following snippet of code?

   You may assume `Random.nextInt(...)` runs in $\Theta(1)$ time.

```
SortedList<Integer> list = new SortedList<>();
Random rand = new Random();
for (int i = 0; i < n; i++) {
  list.insert(rand.nextInt(n));
}
```

Part 2 - Duplicate Values

For the following questions, imagine you are using the `SortedList` class you defined in PA1 to store information about students in CSE 250 throughout the years. You may assume there are n students. When answering, give the most specific runtime bound you can: if a Big-Theta bound exists, give that, otherwise give the tightest upper bound (Big-O) you can. All bounds should be in terms of n. In at most two sentences, explain the runtime you gave. **Answers given without an explanation will receive no credit**.

6. [4 points] What is the runtime of `SortedList.findRef(...)` if your list is storing the final exam grades of all n students. You may assume that final exam grades are integers from 0 to 100.
7. [4 points] What is the runtime of `SortedList.findRef(...)` if your list is storing the UB Person Number of all n students. You may assume person numbers are unique integers.
8. [2 points] Would your answer to either of the previous two questions change if `SortedList` created a new `LinkedListNode` for every single element rather than what it currently does in PA1? If your answer is yes, explain what the new runtime(s) would be and why. If your answer is no, explain why.

Part 3 - Inequalities

For each of the following growth functions, find specific values for constants c and n_0 to prove the requested bound. Show all work for you proof. **Answers given without valid work will receive no credit.**

9. [5 points] Let $f(n) = 3n^3 + 4n + n\log(n^4)$. Prove $f(n) \in O(n^3)$.
10. [5 points] Let $g(n) = 10n\log(2^n) + 5n$. Prove $g(n) \in \Omega(n^2)$.
11. [10 points] Let $h(n) = 42n^5 + 3n^3 + n$. Prove $h(n) \in \Theta(n^5))$.

Summations

1. $\sum_{i=j}^{k} c = (k - j + 1)c$
2. $\sum_{i=j}^{k} (cf(i)) = c \sum_{i=j}^{k} f(i)$
3. $\sum_{i=j}^{k} (f(i) + g(i)) = \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$
4. $\sum_{i=j}^{k} (f(i)) = \left(\sum_{i=\ell}^{k} (f(i))\right) - \left(\sum_{i=\ell}^{j-1} (f(i))\right)$ **(for any** $\ell < j$**)**
5. $\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \ldots + f(k-1) + f(k)$
6. $\sum_{i=j}^{k} f(i) = f(j) + \ldots + f(\ell - 1) + \left(\sum_{i=\ell}^{k} f(i)\right)$ **(for any** $j < \ell \leq k$**)**
7. $\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{\ell} f(i)\right) + f(\ell + 1) + \ldots + f(k)$ **(for any** $j \leq \ell < k$**)**
8. $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$
9. $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Inequalities

1. If you can find some $h(n)$ where $f(n) \leq h(n)$ and $h(n) \leq g(n)$, then $f(n) \leq g(n)$
2. If $f_1(n) \leq g_1(n)$ and $f_2(n) \leq g_2(n)$, then $f_1(n) + f_2(n) \leq g_1(n) + g_2(n)$
3. If you can find some $h(n) \geq 0$ (for all n) where $f(n) \cdot h(n) \leq g(n) \cdot h(n)$, then $f(n) \leq g(n)$
4. Take it as a given that: $\theta(1) \leq \theta(\log(n)) \leq \theta(n) \leq \theta(n^i) \leq \theta(n^j)$ **(for** $i < j$**)** $\leq \theta(2^n)$

Logarithms

1. $\log(n^a) = a \log(n)$
2. $\log(an) = \log(a) + \log(n)$
3. $\log(\frac{n}{a}) = \log(n) - \log(a)$
4. $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
5. $\log(2^n) = 2^{\log(n)} = n$