# CSE 250
## Data Structures

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# Lec 03: Math Refresher

# Announcements and Feedback

- Join Piazza! (Link on course website)
- Academic Integrity Quiz due 2/4 @ 11:59PM **(MUST GET 100%)**
- PA0 due 2/4 @ 11:59PM **(MUST GET 100%)**
- WA1 due 2/4 @ 11:59PM

# Today's Topics

- Summations
- Logarithms
- Limits

# Summations

$$\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + ... + f(k)$$

# Useful Tricks

If ***c*** is a constant (with respect to ***i***)

$$\sum_{i=j}^{k} c = \underbrace{(c + c + \ldots + c)}_{\text{(k - j + 1) times}}$$

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$$\sum_{i=j}^{k} c = (c + c + \ldots + c)$$

$$= (k - j + 1) \cdot c$$

# Useful Tricks

If ***c*** is a constant and ***f*(*i*)** is a function of ***i***:

$$\sum_{i=j}^{k} cf(i) = (cf(j) + cf(j+1) + ... + cf(k))$$

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$$= c(f(j) + f(j+1) + ... + f(k))$$

$$= c\sum_{i=j}^{k} f(i)$$

# Useful Tricks

If ***f*(*i*)** and ***g*(*i*)** are functions of ***i***:

$$\sum_{i=j}^{k} f(i) + g(i) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + ... + (f(k) + g(k))$$

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$$\sum_{i=j}^{k} f(i) + g(i) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + ... + (f(k) + g(k))$$

$$= (f(j) + f(j+1) + ... + f(k)) + (g(j) + g(j+1) + ... + g(k))$$

# Useful Tricks

If ***f***(***i***) and ***g***(***i***) are functions of ***i***:

$$\sum_{i=j}^{k} f(i) + g(i) = (f(j) + g(j)) + (f(j+1) + g(j+1)) + \ldots + (f(k) + g(k))$$

$$= (f(j) + f(j+1) + \ldots + f(k)) + (g(j) + g(j+1) + \ldots + g(k))$$

$$= \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$$

# Useful Tricks

If ***j* < *l* < *k***:

$$\sum_{i=j}^{k} f(i) = f(j) + ... + f(k)$$

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# Useful Tricks

If ***j* < *l* < *k***:

$$\sum_{i=j}^{k} f(i) = f(j) + ... + f(k)$$

$$= f(j) + ... + f(l-1) + f(l) + ... + f(k)$$

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# Useful Tricks

If ***j* < *l* < *k***:

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# Useful Tricks

If ***j*** **<** ***l*** **<** ***k***:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

Subtract to other side

# Useful Tricks

If $j < l < k$:

$$\left(\sum_{i=j}^{k} f(i)\right) = \left(\sum_{i=j}^{l-1} f(i)\right) + \left(\sum_{i=l}^{k} f(i)\right)$$

$$\left(\sum_{i=j}^{k} f(i)\right) - \left(\sum_{i=j}^{l-1} f(i)\right) = \left(\sum_{i=l}^{k} f(i)\right)$$

# Series

Some common closed form solutions:

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

# Summary

- The previous rules will always be provided on WAs and exams
- Usually the goal will be to reduce some complicated summation to a simpler form without a summation
  - Some of the rules get rid of summations
  - Some allow you to manipulate summations/bounds so that you can apply rules that get rid of summations
- Be cognizant of what variables are constant ***with respect to the summation variable*** and which one aren't

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***a*** added together ***n*** times

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# Logarithms

The inverse operation is division

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The inverse operation is ???

# Logarithms

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***a*** multiplied together ***n*** times

The inverse operation is $\mathbf{log_a}$

# Logarithms

$$\log_a(b) = \text{the number of times you multiply a together to get b}$$

$$\log_2(32) = 5$$

$$\log_3(27) = 3$$

$$\log_2(\frac{1}{8}) = -3$$

$$\log_2(2^{10}) = 10$$

# Logarithms

Logarithm is the inverse exponent

$$b^{\log_b(n)} = n = \log_b(b^n)$$

# Product Rule

Let's say $\boldsymbol{n = a \cdot b}$

How are $\mathbf{log_2(}\boldsymbol{n}\mathbf{)}$, $\mathbf{log_2(}\boldsymbol{a}\mathbf{)}$, and $\mathbf{log_2(}\boldsymbol{a}\mathbf{)}$ related?

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How are $\mathbf{log_2(}\boldsymbol{n}\mathbf{)}$, $\mathbf{log_2(}\boldsymbol{a}\mathbf{)}$, and $\mathbf{log_2(}\boldsymbol{a}\mathbf{)}$ related?

$$a = 2 \cdot \ldots \cdot 2 \qquad\qquad b = 2 \cdot \ldots \cdot 2$$

# Product Rule

Let's say $\boldsymbol{n = a \cdot b}$

How are $\mathbf{log_2}(\boldsymbol{n})$, $\mathbf{log_2}(\boldsymbol{a})$, and $\mathbf{log_2}(\boldsymbol{a})$ related?

$$a = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(a) \text{ times}} \qquad b = \underbrace{2 \cdot \ldots \cdot 2}_{\log_2(b) \text{ times}}$$

# Product Rule

Let's say ***n* = *a* · *b***

How are **$\log_2(n)$**, **$\log_2(a)$**, and **$\log_2(a)$** related?

$$a = 2 \cdot ... \cdot 2 \qquad\qquad b = 2 \cdot ... \cdot 2$$

$$n = \underbrace{2 \cdot ... \cdot 2}_{\log_2(n) \text{ times}} = \underbrace{2 \cdot ... \cdot 2}_{\log_2(a) \text{ times}} \cdot \underbrace{2 \cdot ... \cdot 2}_{\log_2(b) \text{ times}}$$

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$$\log_2(n) = \log_2(ab) = \log_2(a) + \log_2(b)$$

# Exponent Rule

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$$= \underbrace{\log_2(a) + \ldots + \log_2(a)}_{n \text{ times}}$$

# Exponent Rule

$$
\begin{aligned}
\log_2(a^n) &= \log_2(a \cdot ... \cdot a) \\
&= \log_2(a) + \log_2(a \cdot ... \cdot a) \\
&= \log_2(a) + \log_2(a) + \log_2(a \cdot ... \cdot a) \\
&= \log_2(a) + ... + \log_2(a) \\
&= n \cdot \log_2(a)
\end{aligned}
$$

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$$= \log_2(a) - \log_2(b)$$

# Change of Base

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$$\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$$

# Summary

| | |
|---|---|
| **Exponent Rule** | $\log(n^a) = a\log(n)$ |
| **Product Rule** | $\log(ab) = \log(a) + \log(b)$ |
| **Division Rule** | $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ |
| **Change of Base** | $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$ |
| **Inverse** | $b^{\log_b(n)} = \log_b(b^n) = n$ |

** for this class, always assume base 2 unless otherwise stated **